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Modeling and analysis of human control actions using fuzzy interactive information systems

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***Abstract*—This paper focuses on the problem of modeling and analyzing the actions of human operators who perform demanding tasks such as real-time control of a complex dynamic plant. To this end, an approach that bases on the idea of interactive fuzzy information systems is proposed. In particular, we discuss the problem of selecting and generating perception and action attributes to describe the aircraft control tasks realized by a skilled pilot. A method consisting of several stages is proposed for modeling pilot-airplane interactions. The initial information system with sensory attributes is replaced with more complex attributes at subsequent stages. To determine the decision rules of the pilot, we apply flow graphs that are suitable for representing fuzzy interactive information systems and for evaluating properties and quality of the human operator's decision model.**

***Keywords*—human assisted control; interactive information systems; fuzzy flow graphs**

I. Introduction

HUMANS have the ability to process, understand and store information in the form that depends on the complexity level of coming data and the involvement of brain structures. A human expert (operator), who performs a control process, observes and causes changes in the state of the controlled object until its desired state is reached.

Many authors were interested in mathematical modeling of pilot control actions from the point of view of control theory. McRuer was one of the first who attempted to describe the human operator's properties. He proposed a mathematical model of the dynamic behavior of a pilot [1]–[3]. This model has a form of a transfer function that contains inertial and differential elements characterized by several time constants that are related to the learned stereotypes, neuromuscular lag, the lead time reflecting the pilot's ability to predict, and the delay between the eye's perception and the brain's response.

In [4], the authors presented the results of experiments conducted on a flight simulator with a group of pilots. The recorded data were used to obtain the average values of the pilot's time constants and the response delay in the McRuer pilot model. Similarly, in [5], different variants of the Tustin-McRuer model were considered and used to approximate pilot behavior.

An interesting concept that helps determine the parameters of pilot behavior was presented in [6]. The authors discussed the structural model by Hess [2], [3], which is a modification of the McRuer's approach. This model takes into account the pilot's ability to close an additional inner loop by responding to the perception of kinesthetic cues. Yet another model presented [6] was the composite model of pilot behavior based on neural network approach.

Other methods such as fuzzy inference, Bayesian networks and Petri nets were described in the [7] for modeling and simulating the behavior of a pilot connected to an avionics system.

In [8], the author discussed the concept of fuzzy control for modeling pilot actions. The control actions generated by a fuzzy pilot model were compared with the control of a human pilot in the task of compensation of the glade slope deviation. In addition, a comparison between the cognitive pilot model and the human pilot was presented.

In contrast to the above-mentioned work on pilot behavior modeling, we consider a decision-oriented approach focusing on the interactions between the pilot and his or her environment. To describe these interactions, the concept of interactive information systems [9], [10] can be used for representing agent-environment connections that emerge in the agent's perception and action processes. An agent is understood as a unit that perceives information derived from an environment and can affect the environment by making his or her actions.

Interactive information systems, and especially interactive decision tables, can be perceived as an extension of the rough set theory introduced by Pawlak [11]. More recently, it has been developed into a generalized paradigm called interactive granular computing [12]–[16]. However, current research on interaction-oriented information systems lacks applications and connections to various approaches that have been introduced in the framework of the rough set and especially fuzzy-rough set theories [17]. To fill this gap and extend our previous work [18], we present in this paper an approach to analysis of control actions of a human operator that combines the concept of interaction information systems with the fuzzy flow graph method [19]. We consider the process of selecting and generating fuzzy attributes in interactive information systems that represent the process of aircraft control. Moreover, we provide an illustrating example basing on data recorded in a real-world application: aircraft altitude stabilization that was performed by a (human) pilot.

The crucial point in applications of the crisp and fuzzy rough set theories consists in constructing a proper information

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system. This is done by identifying the condition and decision attributes, and building a decision table using data collected from a decision-making process. This is not always a trivial task however, especially in the case of a fuzzy knowledge representation, which involves the problem of a suitable choice of fuzzy linguistic values of attributes.

In the presented approach, we propose to distinguish information systems at several stages, that are characterized by different forms of description of the object and its control process.

Initial information about the controlled object, which is collected at the first stage, consist of a set of recorded parameters obtained from measurement sensors and indicators. It serves as a source for generating a more abstract representation at the next stages that has the form of information systems, which depend on the skills and experience of the human operator.

As the process of human reasoning and decision-making is vague in nature, it can be hardly described by using attributes that have merely numeric or symbolic (crisp) values. This is why we prefer to apply fuzzy attributes that adopt values expressed in terms of membership in a set of corresponding linguistic notions that can be suitably selected and tuned to get a more sophisticated and flexible decision model of a human operator.

In the analysis of interactive fuzzy decision tables, the fuzzy flow graph method introduced in [17], [19] is used. Is serves as a tool for determining decision rules of the pilot and to evaluate their quality and statistical characteristics. In this paper, we introduce an improved formal description of the flow graph method, which is clearer and removes some inconsistencies of the original notation.

II. Attributes in Human Operator's Action Model

A. Attributes in Interactive Information Systems

In the interactive information systems [9], [10], one can consider two groups of attributes connected with the perception process, called the atomic and the constructible attributes. Atomic (sensory) attributes represent basic process data, which are obtained with measurement sensors. An atomic attribute could be open, when it is an injection function into its value domain or closed, when it is a surjection function into its value domain, respectively. The information system that possesses only closed type of attributes is called closed static information system [9]. The original information systems introduced by Pawlak are exactly this kind of systems. In our approach to modeling pilot-aircraft interactions, we do not limit ourselves to this kind of systems. The constructible attributes are complex and defined on the basis of atomic attributes. If b is a constructible attribute, then for the defined atomic attributes $a_1, a_2, \ldots, a_n$ and any object x

$$b(x) = F(a_1(x), a_2(x), \ldots, a_n(x)), \tag{1}$$

$$F\colon V_{a_1} \times V_{a_2} \times \ldots \times V_{a_n} \to V_b,$$

where V_{a_i} denotes the value domain of the atomic attributes a_i, V_b is the value domain of the constructible attribute b, and $i = 1, 2, \ldots, n$ [9].

Actions made by an agent will be connected with a special kind of attributes called the action attributes. In [10], the authors suggested that the value of an action attribute should contain, apart from the elementary action, also the specified goal and the expected perceptual results of the given action. This is a consequence of the assumption and observation that every action of the agent bases on the agent's knowledge, the results of perception, and the chosen goal of activity.

B. Modeling the Pilot-Aircraft Interactions

When analyzing the process of controlling an airplane by a pilot, we focus on the following elements:

1) the pilot, who can be seen as an agent with skills and adaptability;
2) the aircraft, which is the closest environment (aircraft states affect the pilot, and his or her actions);
3) the distant environment, e.g., turbulence or weather, that can influence the states of the aircraft and inner states of the pilot, but does not depend on the pilot and the aircraft.

In the following considerations, we focus on the pilot-aircraft interactions, because the distant environment will be indirectly taken into account in the actual states of the pilot and the aircraft. To describe the interactions of the human operator with the controlled object, it is necessary to build abstract information systems of various degree of complexity. We propose to divide the process of creating the human operator action model into several stages.

Stage 1

Information system at the initial stage represents the object (aircraft). It is denoted by $IS^{(1)} = \langle U, A^{(1)}, \{V_a\}_{a \in A^{(1)}} \rangle$ and consists of a universe U of elements characterized by a set of sensory attributes $A^{(1)} = \{a_1, a_2, \ldots, a_n\}$. Values of these attributes $V_{a_i}(T_j)$, $i = 1, \ldots, n$, $j = 1, \ldots, k, \ldots$ are taken from actual measurements using sensors. The parameters $T_1, T_2, \ldots, T_k, \ldots$ indicate instants of time at which the measurements are made.

The set of attributes $A^{(1)} = Env \cup Act$ consists of two groups of attributes:

- Env are the attributes that describe the object (environment), that is, the state variables of the plant;
- Act is the set of attributes representing control variables that can be used by the operator (agent) to interact with the object and change its state.

The values of attributes from the set $A^{(1)}$ are real numbers. This first level information system described above forms the base for creating an interactive information system.

Stage 2

The process of controlling an object involves maintaining specific values (e.g. flight parameters) at the desired level. Let us assume that the operator receives information about the object at the time instant T_k. The state of an object, perceived by the operator, is described on the base of subset of attributes $A^{(1)}$, because the pilot takes into account only those attributes that are important in the actual control task. Hence, a new information system for the object seen by the operator (agent) is described by $IS^{(2)} = \langle U, A^{(2)}, \{V_a\}_{a \in A^{(2)}} \rangle$.

The set of attributes $A^{(2)}$ is useful for creating an inner model of the pilot. The inner model of a human operator plays an essential role in an interaction with a dynamic object. This is understood as a replica of the outside world in the operator's consciousness that constitutes a result of reduction of input information in the perception channels. Only that part of information that is significant for actual activity reaches the operator's consciousness. The inner model bases on selected features and can be treated as a simplified, reduced to characteristic features reflection of the world.

A set of new attributes $A^{(2)}$ bases on the atomic attributes $A^{(1)}$. This is an extension of the set $A^{(1)}$ because it includes additional attributes representing the object's dynamics, which helps the human operator predict its future state. The set of new attributes is denoted by $A^{(2)} = Env_1 \cup Env_2 \cup Act_1$, where:

- Env_1 are attributes representing differences between required and actual values of parameters (control errors);
- Env_2 are attributes that express the direction of change of parameters;
- Act_1 denotes attributes representing the control activities of the human operator.

The values of attributes from the set $A^{(2)}$ are still real numbers, although the ranges of values of these attributes are smaller than those of the attributes from the set $A^{(1)}$. They express deviations and the changes of the variables. We can say that the attributes from the set $A^{(2)}$ are not exactly atomic. They are constructible, since they are functions of the atomic attributes from Env, but in a simple form.

Stage 3

Human operators actually perform the inference process using their inner model. The input of this model is the information from the perception. However, it is transformed and reduced at higher levels of the brain structures into linguistic terms rather than numbers. Therefore, for modeling of such a process, we propose to use information system with fuzzy attributes $IS_{\mathrm{F}} = \langle U, B, \{V_b\}_{b \in B} \rangle$, where $B = Env_{1F} \cup Env_{2\mathrm{F}} \cup Act_{\mathrm{F}} = Env_{\mathrm{F}} \cup Act_{\mathrm{F}}$. The set of attributes B is derived from the set $A^{(2)}$. Hence, the attributes have a constructible form, and the set B consists of:

- $Env_{\mathrm{F}} = Env_{1\mathrm{F}} \cup Env_{2\mathrm{F}}$ – the set of attributes with fuzzy values, describing the perceived object (environment) from the operator's (agent) point of view; they are constructed from the sets Env_1 and Env_2, respectively, and they have the same meaning as mentioned at the previous stage;
- Act_{F} – the set of attributes with fuzzy values, representing an action of the operator (agent), derived directly from the set Act_1.

The values of attributes from the set B are calculated in a step called fuzzyfication, during which the crisp values of each attribute from the set $A^{(2)}$ are transformed into membership degrees to the linguistic values assigned to all attributes from the set B. The main problem, however, is choosing the right form and number of linguistic values associated with each attribute. If the number of linguistic terms is too small, then the granulation of information is not subtle enough. An important issue in the fuzzyfication is an exact location of the limit points of particular membership functions that represent linguistic values.

In practice, experts encounter difficulties, when they try to formalize their knowledge and experience. In consequence, determining the limits points of the fuzzy membership functions becomes a serious problem, especially in the case of large number of input variables. On the other hand, the experts can be supported based on the recorded data set from the classification process they carry out.

Stage 4

At the final stage, the set of attributes is divided into two disjoint parts: a subset of condition and a subset of decision attributes. Hence, an information system can be presented in the form of a decision table. In the case of aircraft control, we can describe the pilot's decision process with the help of an interactive fuzzy information system expressed as a decision table $DT_{\mathrm{F}} = \langle U, C_{\mathrm{F}} \cup D_{\mathrm{F}}, \{V_a\}_{a \in C \cup D} \rangle$. We distinguish the following sets of fuzzy attributes:

- $C_{\mathrm{F}} = Env_{\mathrm{F}} \cup Act_{\mathrm{F}}^{\mathrm{prev}}$ denoting condition attributes that are defined at the previous stage and represent differences between required and actual values of parameters or the direction of parameters changes, where $Act_{\mathrm{F}}^{\mathrm{prev}}$ are attributes that represent previous control actions of the operator;
- $D_{\mathrm{F}} = Act_{\mathrm{F}}^{\mathrm{cur}}$ denoting decision attributes representing current actions and expressing qualitative changes in the control units.

It should be noted that the reasoning and decision-making process performed by the human operator involves inevitable hesitation and delay, especially under stressful conditions. An additional delay can be caused by nonlinearity of the controlled plant and mechanical properties of control elements. Therefore, fuzzy information systems that describe human control of a dynamic object should be specified using condition and decision attributes generated for different moments in time. In our case, decision tables (which are the preferred form of information systems) consists of columns, which represent particular attributes, and rows that correspond to a selected time instants. Due the necessary delay, the decision table DT_{F} include condition attributes at the time T_k and decision attributes at the time T_{k+1} in a selected row.

C. Determining of the Pilot's Action Model

In order to analyze the decision table DT_{F} with fuzzy attributes, it is necessary to set up the initial parameters:

- the starting point of a flight phase;
- the goal of the control process in the form of fixed flight parameters;
- the set of selected attributes $A_{\mathrm{phase}} \subseteq (C_{\mathrm{F}} \cup D_{\mathrm{F}})$ from the decision table DT_{F};
- the size of the time window;
- the number of the time window $i = 1$.

The algorithm for obtaining decision rules for a selected flight phase consists of the following steps, repeated until the end of the flight phase:

1) build a decision table $DT_{\mathrm{F}i}$ from the information system $IS_{\mathrm{F}i}$ for the time window i, with attributes A_{phase};
2) determine the decision rules DR_i by using a fuzzy flow graph representing the decision table $DT_{\mathrm{F}i}$;
3) transfer the decision rules DR_i with additional information such as the certainty factor and strength of particular decision rules to the decision table DT_{phase};
4) select the next time window, $i = i + 1$.

The decision table DT_{phase} includes decision rules DR_i from particular time windows in a given flight phase.

In the next step, the decision rules DR_{phase} for the decision table DT_{phase}, and the rules for the particular actions $DR_{\mathrm{Act}i}$, $i = 1, \ldots, k$, are constructed.

The final decision table contains the knowledge about the object control process.

III. Flow Graph Representation of Fuzzy Information Systems

To construct and analyze fuzzy interactive information systems, we must recall a formal description of a decision table with fuzzy attributes that was introduced in [20].

In the following, a finite set of N elements forming a universe $U = \{u_1, \ldots, u_N\}$ is considered. Every element $u \in U$ is characterized by a tuple of values of fuzzy attributes A_{F} that contains n condition attributes $C_{\mathrm{F}} = \{c_{\mathrm{F}1}, \ldots, c_{\mathrm{F}n}\}$, and m decision attributes $D_{\mathrm{F}} = \{d_{\mathrm{F}1}, \ldots, d_{\mathrm{F}m}\}$.

Any fuzzy attribute is connected with a family of its linguistic values. The subset $\mathbb{C}_{\mathrm{F}i} = \{C_{i1}, \ldots, C_{in_i}\}$ consists of n_i linguistic values of the condition attribute c_{Fi}. and the subset $\mathbb{D}_{\mathrm{F}j} = \{D_{j1}, \ldots, D_{jm_j}\}$, includes m_j linguistic values of the decision attribute $d_{\mathrm{F}j}$, $i = 1, \ldots, n$, and $j = 1, \ldots, m$.

For each element u in the universe U, a degree of membership (which is a value from the interval $[0, 1]$) to particular linguistic values of all condition and decision attributes needs to be assigned. In contrast to crisp information systems, any element $u \in U$ can posses membership (to a degree between 0 and 1) to more than one linguistic value. Because of this, the values of attributes for any $u \in U$ can be interpreted as fuzzy sets defined in the domain of respective linguistic values, as can be observed in Tables I, and II.

In a well defined fuzzy information system, the membership degrees of any element $u \in U$ in the linguistic values $\mathbb{A}_{\mathrm{F}}$ of all fuzzy attributes $A_{\mathrm{F}} = C_{\mathrm{F}} \cup D_{\mathrm{F}}$ must satisfy the following conditions [21]:

$$\begin{aligned} \exists A_{ik} \in \mathbb{A}_{\mathrm{F}i} \quad & (\mu_{A_{ik}}(u) \geq 0.5, \\ & \mu_{A_{ik-1}}(u) = 1 - \mu_{A_{ik}}(u) \ \vee \\ & \mu_{A_{ik+1}}(u) = 1 - \mu_{A_{ik}}(u)), \end{aligned} \tag{2}$$

$$|\mathbb{A}_{\mathrm{F}i}(u)| = \sum_{A_{ik} \in \mathbb{A}_{\mathrm{F}i}} \mu_{A_{ik}}(u) = 1. \tag{3}$$

By considering all combinations of linguistic values, one can generate a complete set of p decision rules, denoted by $DR = \{R^1, \ldots, R^p\}$.

We can express a decision rule $R^k \in DR$ as follows

$$\begin{array}{ll} \text{IF} & [(c_{\mathrm{F}1} \text{ is } C_1^k) \ \ldots \text{ AND } (c_{\mathrm{F}i} \text{ is } C_i^k) \ \ldots \\ & \text{AND } (c_{\mathrm{F}n} \text{ is } C_n^k)] \\ \text{THEN} & [(d_{\mathrm{F}1} \text{ is } D_1^k) \ \ldots \text{ AND } (d_{\mathrm{F}j} \text{ is } D_j^k) \ \ldots \\ & \text{AND } (d_{\mathrm{F}m} \text{ is } D_m^k)], \end{array} \tag{4}$$

where: $k = 1, \ldots, p$,
$p = \left(\prod_{i=1}^{n} n_i\right) \times \left(\prod_{j=1}^{m} m_j\right)$,
$C_i^k \in \mathbb{C}_{\mathrm{F}i}$, $i = 1, \ldots n$,
$D_j^k \in \mathbb{D}_{\mathrm{F}j}$, $j = 1, \ldots, m.$

It should be noted that that real-world decision systems usually contain only a subset of all possible decision rules. By inspecting particular elements $u \in U$ from a given information system, it is possible to check how they support a selected decision rule $R^k \in DR$. It can be done by evaluating the degree of confirmation of the decision rule's premise, and the degree of confirmation of the decision rule's conclusion. To this end, a T-norm operator (`prod`) is used to obtain the resulting membership of a given element $u \in U$ in those linguistic values of attributes that are present in a selected decision rule R^k.

Confirmation degree of the decision rule $R^k \in DR$ by an element $u \in U$, denoted as $\mathrm{cfm}^k(u)$, is defined as follows

$$\mathrm{cfm}^k(u) = \mathrm{T}(\mathrm{cfm}_{\mathrm{P}}^k(u), \mathrm{cfm}_{\mathrm{C}}^k(u)), \tag{5}$$

where $\mathrm{cfm}_{\mathrm{P}}^k(u)$ is the degree of confirmation of the decision rule's premise

$$\mathrm{cfm}_{\mathrm{P}}^k(u) = \mathrm{T}(\mu_{C_1^k}(u), \mu_{C_2^k}(u), \ldots, \mu_{C_n^k}(u)), \tag{6}$$

and $\mathrm{cfm}_{\mathrm{C}}^k(u)$ is the degree of confirmation of the decision rule's conclusion

$$\mathrm{cfm}_{\mathrm{C}}^k(u) = \mathrm{T}(\mu_{D_1^k}(u), \mu_{D_2^k}(u), \ldots, \mu_{D_m^k}(u)). \tag{7}$$

Basing on the formulae (6), (7) and (5), we are able to determine for all elements $u \in U$ a detailed support of the decision rule $R^k \in DR$:

support of the premise

$$\begin{aligned} \mathrm{supp}(\mathrm{cfm}_{\mathrm{P}}^k) = \{ & \mathrm{cfm}_{\mathrm{P}}^k(u_1)/u_1, \ \mathrm{cfm}_{\mathrm{P}}^k(u_2)/u_2, \ \ldots, \\ & \mathrm{cfm}_{\mathrm{P}}^k(u_N)/u_N\}, \end{aligned} \tag{8}$$

support of the consequent

$$\begin{aligned} \mathrm{supp}(\mathrm{cfm}_{\mathrm{C}}^k) = \{ & \mathrm{cfm}_{\mathrm{C}}^k(u_1)/u_1, \ \mathrm{cfm}_{\mathrm{C}}^k(u_2)/u_2, \ \ldots, \\ & \mathrm{cfm}_{\mathrm{C}}^k(u_N)/u_N\}, \end{aligned} \tag{9}$$

and support of the decision rule R^k, respectively

$$\begin{aligned} \mathrm{supp}(R^k) = \{ & \mathrm{cfm}^k(u_1)/u_1, \ \mathrm{cfm}^k(u_2)/u_2, \ \ldots, \\ & \mathrm{cfm}^k(u_N)/u_N\}. \end{aligned} \tag{10}$$

The support sets defined in (8), (9), and (10) are used in the generalized concept of flow graphs that is suitable for modeling and analysis of fuzzy information systems.

The idea of a crisp flow graph that helps to evaluate the statistical properties of crisp decision systems, was proposed by Pawlak [22]–[24]. The fuzzy-oriented flow graph approach was introduced in [17], [19].

TABLE I
DECISION TABLE FOR TIME WINDOW 1

	$c_{\mathrm{F}1}$					$c_{\mathrm{F}2}$			$c_{\mathrm{F}3}$			$c_{\mathrm{F}4}$			$d_{\mathrm{F}1}$		
	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	C_{41}	C_{42}	C_{43}	D_{11}	D_{12}	D_{13}
u_1	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0
u_2	0.0	0.0	0.9	0.1	0.0	0.0	0.9	0.1	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1
u_3	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1	0.0	1.0	0.0
u_4	0.0	0.0	0.1	0.9	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.2	0.8
u_5	0.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1	1.0	0.0	0.0	0.0	0.2	0.8	0.2	0.8	0.0
u_6	0.0	0.0	1.0	0.0	0.0	0.0	0.8	0.2	0.0	1.0	0.0	0.2	0.8	0.0	0.0	1.0	0.0
u_7	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0
u_8	0.0	0.0	0.2	0.8	0.0	0.1	0.9	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.1	0.9	0.0
u_9	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.1	0.9	0.0	0.0	1.0	0.0
u_{10}	0.0	0.0	0.0	0.1	0.9	0.0	0.0	1.0	0.0	0.1	0.9	0.0	1.0	0.0	0.0	0.0	1.0

TABLE II
DECISION TABLE FOR TIME WINDOW 2

	$c_{\mathrm{F}1}$					$c_{\mathrm{F}2}$			$c_{\mathrm{F}3}$			$c_{\mathrm{F}4}$			$d_{\mathrm{F}1}$		
	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	C_{41}	C_{42}	C_{43}	D_{11}	D_{12}	D_{13}
u_1	0.0	0.0	0.0	0.9	0.1	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0
u_2	0.1	0.9	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
u_3	0.0	0.0	1.0	0.0	0.0	0.0	0.8	0.2	0.0	0.0	1.0	1.0	0.0	0.0	0.1	0.9	0.0
u_4	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1	0.1	0.9	0.0	0.0	1.0	0.0
u_5	0.0	0.2	0.8	0.0	0.0	0.1	0.9	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0
u_6	0.0	0.0	0.9	0.1	0.0	0.0	0.9	0.1	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1
u_7	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.9	0.1	1.0	0.0	0.0
u_8	0.0	0.8	0.2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	1.0	0.0
u_9	0.2	0.8	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.8	0.2	0.0
u_{10}	0.0	0.1	0.9	0.0	0.0	0.0	0.9	0.1	0.0	0.2	0.8	0.8	0.2	0.0	0.0	1.0	0.0

A crisp flow graph [23], [24] has the form of directed acyclic final graph $G = (\mathcal{N}, \mathcal{B}, flow)$, where $\mathcal{N}$ is a set of nodes, $\mathcal{B} \subseteq \mathcal{N} \times \mathcal{N}$ is a set of directed branches, $flow\colon \mathcal{B} \to \mathrm{R}^+$ is a flow function with values in the set of positive real numbers.

For a given pair $(A, B) \in \mathcal{B}$, the node A is an input of the node B, and the node B is an output of the node A. The throughflow from the node A to the node B is denoted by $flow(A, B)$.

The input and the output of the node A are denoted by $In(A)$ and $Out(A)$, respectively.

The set $In(G)$ of the input nodes of a graph G is defined as

$$In(G) = \{A \in \mathcal{N}\colon\ In(A) = \emptyset\}\,, \tag{11}$$

and the set $Out(G)$ of the output nodes of G as

$$Out(G) = \{A \in \mathcal{N}\colon\ Out(A) = \emptyset\}\,.$$

Inflow of the node $A \in \mathcal{N}$ is determined by summing the throughflow from its input

$$flow_+(A) = \sum_{B \in In(A)} flow(B, A)\,, \tag{12}$$

and outflow of the node A by summing the throughflow to its output

$$flow_-(A) = \sum_{B \in Out(A)} flow(A, B)\,. \tag{13}$$

Since for any internal node A its inflow $flow_+(A)$ is equal to its outflow $flow_-(A)$, it is called flow of the node A.

The flow in the graph G can be obtained by summing the inflow from the graph input nodes or by summing the outflow to the graph output nodes, as follows

$$flow(G) = \sum_{A \in In(G)} flow_-(A) = \sum_{A \in Out(G)} flow_+(A)\,. \tag{14}$$

To make it easier to compare the flow of different nodes in a graph, flow normalization should be performed.

The normalized throughflow from the node A to the node B is defined as

$$Flow(A, B) = \frac{flow(A, B)}{flow(G)}\,, \tag{15}$$

and the normalized flow of the node A as

$$Flow(A) = \frac{flow(A)}{flow(G)}\,. \tag{16}$$

Flow graphs can be used for representing information systems that are expressed in the form of decision tables. This is done by constructing a layered feedforward graph consisting of a set of condition layers and a set of decision layers. Every graph layer contains nodes corresponding to values of one single attribute. The first (arbitrarily chosen) condition attribute is represented by the input layer and the remaining condition attributes by consecutive hidden layers of nodes in the flow graph. Although, in the general case, there may be several decision attributes present in a decision system, it is sufficient to consider only one decision attribute. Therefore, the output layer containing nodes corresponding to the decision attribute constitutes the final part of the flow graph.

Since each element $u \in U$ is described by a combination of attribute values, we can imagine it "flowing" by taking a distinct path consisting of the corresponding nodes in the condition and decision layers of the flow graph. However, in the case of fuzzy attributes, each element $u \in U$ can possess several linguistic values simultaneously, and therefore it may "flow" through more than one path in the flow graph.

A selected path in the flow graph corresponds to a single row in the decision table and generates a respective decision rule. To determine consistency of a decision rule, it is necessary to check whether there are no other decision rules that have the same condition attribute values, but different values of decision attributes. In terms of flow graph representation, this can be done by determining a certainty factor defined for a branch of a flow graph G as follows

$$\mathrm{cer}(A,B) = \frac{Flow(A,B)}{Flow(A)} \,. \tag{17}$$

It is also possible to evaluate the contribution of different decision rules to a selected decision by determining the coverage factor that is defined for a branch of a flow graph G by

$$\mathrm{cov}(A,B) = \frac{Flow(A,B)}{Flow(B)} \,. \tag{18}$$

The certainty and coverage factors satisfy the following balance equations

$$\sum_{B \in Out(A)} \mathrm{cer}(A,B) = 1 \,, \qquad \sum_{A \in In(B)} \mathrm{cov}(A,B) = 1 \,. \tag{19}$$

To determine the flow between selected nodes of neighbouring graph layers it is necessary to take into account the degree of membership of particular elements of the universe U in the linguistic values represented by those nodes. For a given node A, we denote by $\widetilde{A}$ a fuzzy set in the domain U that expresses the membership of the elements $u \in U$ in a linguistic value that is represented by the node A.

Let us consider a branch between the nodes A and B. The cardinality of intersection of the sets $\widetilde{A}$ and $\widetilde{B}$, is equal to the fuzzy throughflow $flow(A,B)$ for the branch (A,B). The following balance equation holds for the nodes of the input and internal layers, if the fuzzy sets intersection is determined using the T-norm operator prod:

$$flow_{-}(A) = |\widetilde{A}| \,, \tag{20}$$

$$|\widetilde{A}| \;=\; \sum_{B \in Out(A)} flow(A,B) \;=\; \sum_{B \in Out(A)} |\widetilde{A} \cap \widetilde{B}| \,.$$

Under the same assumption the balance equation for the nodes of the output and internal layers,

$$flow_{+}(A) = |\widetilde{A}| \,, \tag{21}$$

$$|\widetilde{A}| = \sum_{B \in I(A)} flow(B,A) = \sum_{B \in I(A)} |\widetilde{A} \cap \widetilde{B}| \,.$$

Furthermore, the equality $flow_{+}(A) = flow_{-}(A) = flow(A)$ is satisfied for any internal node A.

For a more concise notation of the dependencies in a flow graph, its initial part consisting of the input and the hidden layers, which represent condition attributes, can be replaced by a single layer of nodes. Such a resulting layer consists of the nodes that correspond to different combinations of the condition attributes' linguistic values.

Let us denote by A^* a node in the resulting layer. As a selected combination of linguistic values of condition attributes, the node A^* expresses the premise of some decision rule R^k. Support of the premise of the rule R^k can be calculated by using (8). If a node B in the output layer expresses a conclusion of the decision rule R^k, then the branch (A^*,B) represents this decision rule.

In consequence, by applying the definition (10), we can express the fuzzy cardinality of the support of the rule R^k as the flow between the node A^* and the node B

$$|\mathrm{supp}(R^k)| = flow(A^*, B) \,. \tag{22}$$

Finally, basing on the formulae (8), (9), and (10), the certainty factor $\mathrm{cer}(A^*,B)$, the coverage factor $\mathrm{cov}(A^*,B)$, and the strength $\mathrm{str}(R^k)$ of the decision rule R^k can be determined, as follows

$$\mathrm{cer}(R^k) = \frac{|\mathrm{supp}(R^k)|}{|\mathrm{supp}(\mathrm{cfm}_{\mathrm{P}}^k)|} = \mathrm{cer}(A^*,B) \,, \tag{23}$$

$$\mathrm{cov}(R^k) = \frac{|\mathrm{supp}(R^k)|}{|\mathrm{supp}(\mathrm{cfm}_{\mathrm{C}}^k)|} = \mathrm{cov}(A^*,B) \,, \tag{24}$$

$$\mathrm{str}(R^k) = \frac{|\mathrm{supp}(R^k)|}{|U|} = Flow(A^*,B) \,. \tag{25}$$

All assumptions and requirements made in the above definitions are necessary to satisfy flow conservation equations in a fuzzy flow graph.

IV. Example

To illustrate the presented approach in analysis of pilot actions, we consider the task of maintaining the aircraft altitude at a desired level.

The subset of condition attributes includes:

c_{F1} – difference between desired and actual altitude,
c_{F2} – vertical speed,
c_{F3} – derivative of vertical speed,
c_{F4} – previous change of rudder deflection.

The subset of decision attributes includes a single element:

d_{F1} – change of rudder deflection.

The attributes have the following sets of linguistic values:

$C_{\mathrm{F1}} = \{C_{11}$ – "Large Negative", C_{12} – "Small Negative", C_{13} – "Zero", C_{14} – "Small Positive", C_{15} – "Large Positive"$\}$,

$C_{\mathrm{F2}} = \{C_{21}$ – "Negative", C_{22} – "Zero", C_{23} – "Positive"$\}$,

$C_{\mathrm{F3}} = \{C_{31}$ – "Negative", C_{32} – "Zero", C_{33} – "Positive"$\}$,

$C_{\mathrm{F4}} = \{C_{41}$ – "Negative", C_{42} – "Zero", C_{43} – "Positive"$\}$,

$D_{\mathrm{F1}} = \{D_{11}$ – "Decrease", D_{12} – "Zero", D_{13} – "Increase"$\}$.

Membership functions of all linguistic values of the condition and decision attributes have a standard triangular and trapezoidal form. Tables I and II correspond to decision tables $DT_{\mathrm{F}1}$ and $DT_{\mathrm{F}2}$ (subsection II-C) from the first and the second time windows, respectively.

All fuzzy attributes can be represented by subsequent layers of nodes in the respective fuzzy flow graph. A single node corresponds to a linguistic value of a selected attribute. The layers representing the condition attributes can be replaced by a single (resulting) layer. Taking into account all combinations of linguistic values, we obtain all possible decision rules and determine strength, certainty and the coverage factors for them. All decision rules having certainty factor below 0.6, coverage factor below 0.1, and strength below 4% are discarded.

As described in the subsection II-C, we get for the first time window the set of decision rules DR_1 that corresponds to a fuzzy flow representing the decision table $DT_{\mathrm{F}1}$:

R_1^1: IF [($c_{\mathrm{F}1}$ is C_{13}) AND ($c_{\mathrm{F}2}$ is C_{22}) AND ($c_{\mathrm{F}3}$ is C_{32}) AND ($c_{\mathrm{F}4}$ is C_{42})] THEN ($d_{\mathrm{F}1}$ is D_{12}),

R_1^2: IF [($c_{\mathrm{F}1}$ is C_{14}) AND ($c_{\mathrm{F}2}$ is C_{22}) AND ($c_{\mathrm{F}3}$ is C_{31}) AND ($c_{\mathrm{F}4}$ is C_{43})] THEN ($d_{\mathrm{F}1}$ is D_{12}),

R_1^3: IF [($c_{\mathrm{F}1}$ is C_{14}) AND ($c_{\mathrm{F}2}$ is C_{23}) AND ($c_{\mathrm{F}3}$ is C_{33}) AND ($c_{\mathrm{F}4}$ is C_{42})] THEN ($d_{\mathrm{F}1}$ is D_{13}),

R_1^4: IF [($c_{\mathrm{F}1}$ is C_{15}) AND ($c_{\mathrm{F}2}$ is C_{23}) AND ($c_{\mathrm{F}3}$ is C_{33}) AND ($c_{\mathrm{F}4}$ is C_{42})] THEN ($d_{\mathrm{F}1}$ is D_{13}).

For the second time window, we determine the set of decision rules DR_2:

R_2^1: IF [($c_{\mathrm{F}1}$ is C_{12}) AND ($c_{\mathrm{F}2}$ is C_{21}) AND ($c_{\mathrm{F}3}$ is C_{31}) AND ($c_{\mathrm{F}4}$ is C_{42})] THEN ($d_{\mathrm{F}1}$ is D_{11}),

R_2^2: IF [($c_{\mathrm{F}1}$ is C_{12}) AND ($c_{\mathrm{F}2}$ is C_{21}) AND ($c_{\mathrm{F}3}$ is C_{33}) AND ($c_{\mathrm{F}4}$ is C_{41})] THEN ($d_{\mathrm{F}1}$ is D_{12}),

R_2^3: IF [($c_{\mathrm{F}1}$ is C_{13}) AND ($c_{\mathrm{F}2}$ is C_{22}) AND ($c_{\mathrm{F}3}$ is C_{32}) AND ($c_{\mathrm{F}4}$ is C_{42})] THEN ($d_{\mathrm{F}1}$ is D_{12}),

R_2^4: IF [($c_{\mathrm{F}1}$ is C_{13}) AND ($c_{\mathrm{F}2}$ is C_{22}) AND ($c_{\mathrm{F}3}$ is C_{33}) AND ($c_{\mathrm{F}4}$ is C_{41})] THEN ($d_{\mathrm{F}1}$ is D_{12}),

R_2^5: IF [($c_{\mathrm{F}1}$ is C_{14}) AND ($c_{\mathrm{F}2}$ is C_{21}) AND ($c_{\mathrm{F}3}$ is C_{31}) AND ($c_{\mathrm{F}4}$ is C_{43})] THEN ($d_{\mathrm{F}1}$ is D_{12}).

By using the formulae (23), (24), and (25), the statistical characteristics of the decision rules for every time window can be discovered.

In the first time window, the decision rule R_1^1 has the highest strength, and the decision rule R_1^4 has the lowest, i.e., $\mathrm{str}(R_1^1) = 41.7\%$, and $\mathrm{str}(R_1^4) = 8.1\%$, respectively. Certainty factors are relatively high for all rules: $\mathrm{cer}(R_1^1) = 0.98$, $\mathrm{cer}(R_1^2) = 0.85$, $\mathrm{cer}(R_1^3) = 0.91$, and $\mathrm{cer}(R_1^4) = 1.00$. The decision rule $\mathrm{str}(R_1^3)$ has the highest coverage factor and the decision rule $\mathrm{str}(R_1^2)$ has the lowest: $\mathrm{cov}(R_1^3) = 0.62$, and $\mathrm{cov}(R_1^2) = 0.18$, respectively.

By merging the sets DR_1 and DR_2, a new decision table (DT_{phase}) containing these representative decision rules can be obtained, which can be treated as a part of the model of pilot's actions. Furthermore, we can also select decision rules with respect to particular linguistic values of decision attribute, denoted by $DR_{d_1 i}, i = 1, 2, 3$. It is also possible to detect inconsistent decision rules in the Table DT_{phase}. In such a case, expanding the set of condition attributes may be necessary.

In the presented example, we consider only two time windows for extracting decision rules of a pilot. In reality, the analysis of a large number of decision tables obtained from control protocols for a given flight phase, is needed. In this way, we get closer to an exact action model of an operator.

V. Conclusions

The concept of interactive information systems is suitable for modelling control processes performed on a complex plant by a human expert. The considered task of stabilization of the aircraft's altitude is a dynamic process, which requires repeated recording and analysis of the pilot's control actions. Information systems obtained for different time windows can be represented by fuzzy flow graphs for determining the decision rules of the pilot. The presented approach can be developed in future work by considering more linguistic values of fuzzy attributes to obtain improved models of pilot actions.

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