Algorithms for reliable networks deployment in mesh topology with flow control

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Abstract — the article presents innovative methods for designing and controlling topology in mesh networks. The authors of the paper simulate the procedure of deploying a mesh structure and prove that the shortest path for such a network is also one with the least number of hops in the sense of intermediate nodes.

Keywords — mesh, network, planning

I. INTRODUCTION

THE global growth of computer networks forces Internet service providers to guarantee quality and continuity of services. The network carriers in turn force the network designers to create systems of high dependability [1], [4]. In the case of wireless networks (mesh networks in particular) it is a principle that connections cannot cross [2] which implies that the spanning graph that forms it must be planar. The authors of this article prove that there exists a network-spanning algorithm with dependability of at least two (i.e. one with the minimum of two independent non-crossing connections). In order to verify this assumption a network-dependability criterion can be formulated ([1], [4]): "for a given network calculate an incidence matrix first, next determine the flow between all its nodes according to Ford-Fulkerson's algorithm [3]". The value of the maximum flow defines the number of alternative connections between network nodes. If this value for a given pair of nodes equals 2 it is evident that one of the intermediate node fails there still exists one more alternative route detouring the failed node.

II. MODELING WIRELESS MESH NETWORKS

Any mesh sensor network can be modeled with a directed graph defined in [3] as an ordered pair $<L,K>$ where L is a set of nodes, K is a set of vertex pairs ordered and defined on the set K.

An ordered pair $<x,y>$ belonging to K is called a directed arc where nodes x and y denote its beginning and the end, respectively. Now, if K is a set of unordered pairs called undirected arcs then the graph G=$<K,L>$ is called undirected.

If the set K contains both directed and undirected arcs then the graph G=$<K,L>$ is called a mixed graph. Undirected and mixed graphs can be treated as a particular case of directed graph since every undirected arc can be replaced with a pair of arcs directed in opposite directions. The graphs in which to each pair of nodes at most one interconnecting directed arc has been assigned are called unigraphs.
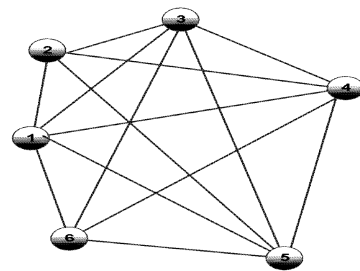

Fig. 1. An example of a network with high dependability.

This definition makes it possible to create a space of solutions for the problem of modeling a mesh network, a sensor network being a typical example. It is noteworthy that the nodes of the graph will now be represented by sensors while its arcs will assume a form of wireless connections between them.

A thus modeled space of solutions allows one to apply various optimization algorithms for communication between sensors. A particular heed must be taken of the values assigned to weights on the graph arcs, which can be interpreted arbitrarily according to the needs. For example, a weight can symbolize throughput achievable between network nodes or pathloss, or distance etc.

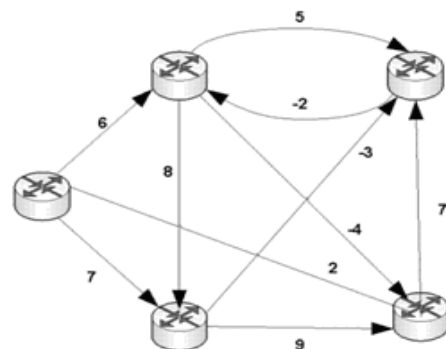For a mobile mesh network one more constraint should be imposed regarding the radio medium.

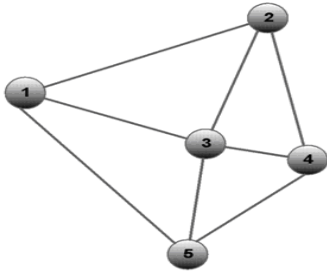Fig. 2. An example of a dispersed (mesh) network

Fig. 3. An example of a proper graph of interconnections for a mesh network

The constraint says that interconnections between the network nodes must not cross due possibly excessive interference in which it might otherwise result [2]. The authors have solved the problems utilizing basic geometrical constraints regarding distances between nodes.

A simple recipe for a planar graph has been provided by means of a triangulation method.

The authors have solved the problem employing the constraints that apply to dependability in computer networks [1], [4]. For modeling purposes it was assumed that the weight assigned to arcs represent pathloss (see chapter 3 for details) whereas nodes are represented by transmitters forming a mesh structure.

III. PROPAGATION MODELING

In wireless systems planning beside the purely mathematical considerations regarding optimal data routing it is crucial to properly predict the radio wave attenuation between intervening nodes. The physical distance, the operational frequency and the type of environment play a leading role in the planning process. Since the target systems for the procedures described in this paper are both FWA (Fixed Wireless Access) and WSN (Wireless Sensor Network) it is important to select an appropriate propagation model among galore of available models found in literature. As for FWA systems (e.g. WiMAX) three models appear to be adequate candidates: the One-Slope model (OS), the Three-Slope model (3S) and the Stanford University Interim model (SUI).

The OS model represents the simplest approach to the propagation modeling assuming monotonous diminishing of radiated power along the distance from BS (base station) as in (1).

$$L(f,d,n) = L_0(f,d_0) + n \cdot 10\log\left(\frac{d}{d_0}\right) + X_\sigma \qquad (1)$$

where $L_0(f,d_0)=20log(c/4\pi d_0 f)$, X_σ – log-normal random variable, d_0 – reference distance, c – speed of light, f – frequency, n – power decay coefficient (assumed equal 2).

The three-slope model is a recommended approach for designing mesh structures in IEEE 802.16 family, according to [6]. Its version for macrocells (which is the case in FWA planning considered herein) is given by (2).

The 3S model is in fact a more elaborate version of OS model in that it assumes that as the receiver is moved away from BS the power decay n coefficient increases as well: $n=2$

for distances shorter than 50 m, $n=4$ for $d>500$ m and $n=3$ in the interim region. No other distinction between types of propagation environment is provided.

$$L = 20\log(4\pi f / c) + 20\log(50) + 30\log(500/50) + $$
$$+ 40\log(d/500) = 40\log(d) - 1 \qquad \forall d > 500m \qquad (2)$$

The third propagation model assumed is the SUI formula [8] containing a set of environmental parameters (variables a, b and c which assume values depending on the chosen terrain type – A, B or C and thus affect the power decay coefficient n) which render the outcomes sensitive to the environment in which propagation takes place (despite rather coarse discrimination). Type A terrain (maximum path loss) is hilly with moderate-to-heavy tree densities, type C is mostly flat terrain with light tree densities and type B capturing intermediate path loss conditions. The path loss is calculated with (3).

$$L = 20\log\left(\frac{4\pi d_0 f}{c}\right) + 10 \cdot n \cdot \log\left(\frac{d}{d_0}\right) + s \qquad \text{for } d > d_0 \qquad (3)$$

where $d_0=100$ m, n – power decay coefficient with $n=(a-b \cdot h_{TX}+c/h_{RX})$, h_{TX} and h_{RX} being the transmitter and receiver heights, respectively. The comparison of results obtained for $f=3.5$ GHz (typical operation frequency for WiMAX) is shown in fig. 4.

In the simulation program presented in this work, it is left to the user's discretion which of the three models is selected. Regardless of the choice some common requirements must be met when polling locations for BS deployment at a given area (usually a square with a side length of a few km):

1. a minimum node-to-node distance is arbitrarily chosen by the user (e.g. a few hundred meters) to avoid unrealistic proximity unacceptable for both interference and economic reasons;

2. each node must be located within the radio visibility range of at least two other nodes to guarantee minimum network survivability;

3. The maximum distance dmax (or, equivalently, the maximum path loss L_{max}) between a given node and the two nodes described in condition 2. must be such that some minimum system performance level is sustained.

It can be computed by subtracting the minimum required power at the receiver input (P_{RX}) from the equivalent radiated power (ERP) at the BS antenna augmented by the receiver antenna gain G_{Rx} ($L_{max}=ERP-P_{RX}+G_{RX}$). The resulting value determines the maximum attenuation between BS and the receiver station necessary to uphold successful transmission at the lowest performance level. For example, in WiMAX the absolutely minimum conditions for maintaining a connection require $P_{RX}=-94$ dBm (assuming the receiver Noise Figure $NF=7$ dB, and the implementation margin of 5 dB) according to [7].

As it can be observed in each of the above models, the major aim is to find the power decay coefficient n matching closely the type of the intervening propagation environment.
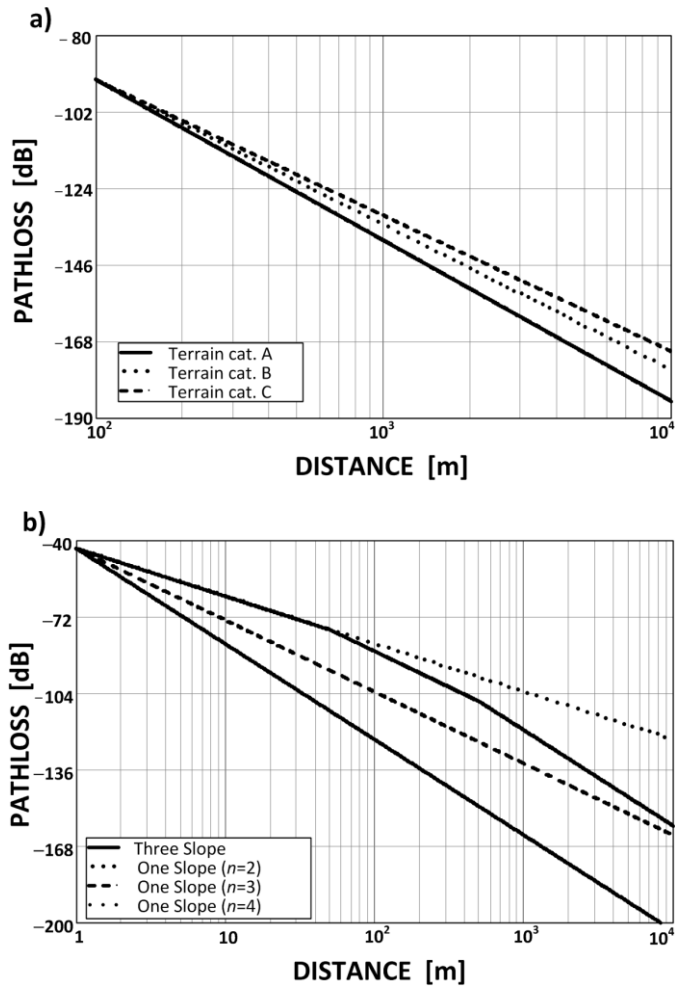
Fig. 4. Pathloss with different propagation models: a) OS and 3S; b) SUI

In WSN networks it may occur that both the receiving and transmitting party are located directly on the ground or at a very low altitude (for instance in Smart Dust project or some military applications) as opposed to FWA systems where one tends to fix the BS antennas at possibly high locations.

In such case at least one half of the first Fresnel's zone (which degree of clearance determines the visibility) will be obstructed by the ground (not mentioning additional obstructions such as vegetation or local ground protrusions) – if so, according to research and measurements performed in [9], the n coefficient should be assumed equal to 4 or more (if additional obstructions are present).

This result yields 40 dB pathless per decade of distance in low-antenna-height WSN networks.

IV. DESCRIPTION OF THE SOLUTION SPACE

Chapter 2 indicates that the space of solutions will encompass a set of acute triangles i.e. such where the cosine of all angles is greater than zero. Oppositely, for all triangles with one angle cosine being less than zero, the longest side is assumed to be nonexistent since it is burdened with an excessive connection cost (in terms of, for instance, large pathloss). Furthermore, the longest side may cause some of the

graph arcs to cross, which is a prohibited situation (refer to chpt. 1).

Obviously, if for so formulated space the network does not satisfy the necessary condition to fulfill the dependability criterion (i.e. the exception in fig. 3), the shortest graph branch (arc) should be connected to close it.

In order to formalize the space of possible solutions the authors have written a function that limits this space. A matrix form with weights has been used to describe the graph [3].

V. ESTIMATING THE COMPUTATIONAL COMPLEXITY FOR THE „THREE NEIGHBORS" PROBLEM

Based on [3] the authors have estimated the computational complexity of the proposed algorithm. It can be implemented in the form of three embedded "for" loops. The crucial point in the analysis was to analyze the triple "for" loop. It suffices to notice that the three loops are in a specific mutual relation, namely the number of iteration of any inner loop is smaller than that of the outer loop. Formally, this can be written in the form:

```
for i=(1:n)
        for j=(i:n)
                for m=(j:n)
                (...)
                endfor
        endfor
endfor
```

Since i, j, m are of the same n-th order it means that the algorithm computational complexity equals (4).

$$O\left(\binom{n}{3}\right) = O\left(\frac{n(1+n)(2+n)}{6}\right) = O(n^3) \qquad (4)$$

This result indicates that the algorithm performs in a polynomial time and the networks planning time is always solvable in a finite time.

VI. EXAMPLES OF SOLUTIONS

In order to verify the procedure described herein the authors have developed a simulator fulfilling the assumptions presented in points 1-4.

In fig. 5 an example of mobile nodes randomly deployed to form a mesh network.

The node pairs (6;10) and (3;4) are, respectively, the closest and the farthest spaced pairs in the whole network. After calculations a final topology has been achieved as shown in fig. 6. Next the authors have verified whether the computed topology satisfies the condition of the dependability theorem [2], [4].

For further reassurance the authors have carried out a few hundred simulations and verified them against the dependability theorem obtaining positive results in each test.
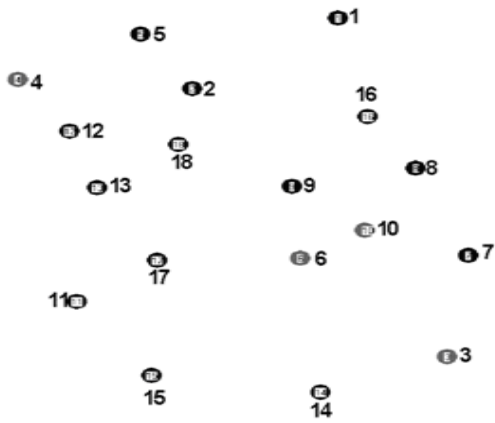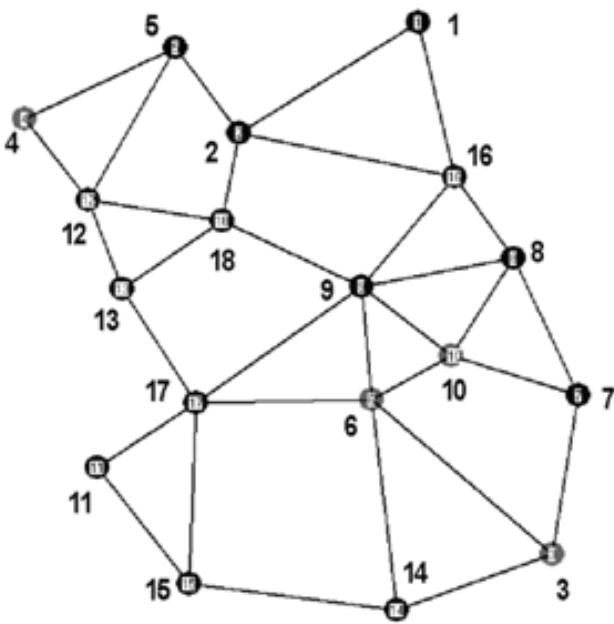
Fig. 5. Example of a network before optimization

Fig. 6. Reliable network of minimum intra-system interference

Fig. 7. Matrix representation of a reliable network depicted in fig. 6

VII. METHODS FOR DETERMINING FLOWS IN MESH NETWORKS

Historically and presently four methods have been applied to determine flows in networks:

- Flow Assignment (FA),
- Capacity Assignment (CA),
- Capacity and Flow Assignment (CFA),
- Topology Capacity and Flow Assignment (TCFA).

Basic criteria used in network planning include: average packet delay, network deployment cost, throughput, values of reliability factors, network maintenance costs [11], [13]. Another issue relates to the fact that present methods for the flow determination rely on criteria associated to the costs of leased lines and their length (albeit practically with the link throughput). This problem has been addressed in [14], [15].

A. FLOW TYPES IN NETWORS

There are three kinds of flows identified in literature [11], [12] that can be encountered in mesh (computer) networks:

- single component flow,
- multi-component flow without ramifications,
- multi-component flow with ramifications.

1) SINGLE COMPONENT FLOW

A network S is given whose structure is determined by a proper and oriented unigraph $G=<N,L>$ (fig. 8). Let s and u be the nodes of a set N. Any function f can be called a static flow of value v from the node s to the node u in the network S if it relates a set L of arcs to the set of nonnegative real numbers which values $f(x,y)$ assigned to particular arcs $<x,y>\in L$ fulfill the equation (5) and the linear inequality (6):

$$\sum_{y\in A(x)} f(x,y) - \sum_{y\in B(x)} f(y,x) = \begin{cases} v & for\ x = s \\ -v & for\ x = u \\ 0 & elsewhere \end{cases} \quad (5)$$

for every $x\in N$.

$$f(x,y)\geq 0 \quad \underset{\langle x,y\rangle\in L}{\forall} \quad (6)$$

where: $A(x) = \{y: y \in N \wedge <x, y> \in L\}$ is the set of network nodes which are sinks to the arcs reaching from the node x, whereas $B(x) = \{y: y \in N \wedge <y, x> \in L\}$ is a set of network nodes which are sources to arcs reaching out of the node x. The value $f(x,y)$ is called a flow $<x,y>$ in the arc. The node s is called the source node (or simply – the source), the node u is the target node (or a sink). Constraints (5) are called the equations of flow maintenance in nodes, whereas the constraints (6) are associated with nonnegative nature of flows.

2) MULTI-COMPONENT FLOW WITHOUT RAMIFICATIONS

Let us now define a multi-component flow (fig. 9) which will correspond to the average packed flow in a computer network at a given time interval.
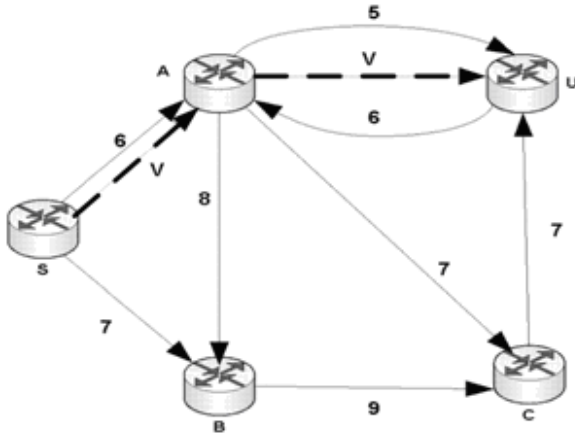
Fig. 8. Single component flow

A component is defined to be a set of packets possessing the same source node and the same sink.

Let r'_{ij} denote the average packet flow intensity directed from the node i do the node j expressed in terms of packets/second, and let $1/\mu$ denote the average packet length in bits/second. Hence, $r_{ij} = r'_{ij}/\mu$ is the average bit stream directed from the node i do the node j in bits/second. Let now $R=[r_{ij}]_{nxn}$ stand for the matrix of elements r_{ij} also called the matrix of intensities injected from the outside. Let all the components from 1 to q be numbered. A pair of nodes s_k and u_k (source and sink) is associated with any k-th component.

Let also r_k denote the average intensity of the k-th component i.e. $r_k = r_{ij}$,where $i=s_k$,$j=u_k$; r_k is also called the value of the k-th component.

Let S be a network, whose structure is defined by a proper and oriented unigraph $G=<N,L>$. The multi-component flow in the network S representing the matrix R of externally injected intensities can be associated to the set of functions f^k ; $L \rightarrow R^+ \cup \{0\}$ $k=1,..., q$, which values $f^k(x,y)$ $k=1,...,q$ assigned to the arcs $<x,y> \in L$ fulfill the set of conditions (7) and (8).

$$\sum_{y\in A(x)} f^k(x,y) - \sum_{y\in B(x)} f^k(y,x) = \begin{cases} r_k & for\ x = s_k \\ -r_k & for\ x = u_k \\ 0 & elsewhere \end{cases} \quad (7)$$

And for each $x \in N, k = 1, ... , q$:

$$f^k(x,y) \geq 0 \underset{\langle x,y\rangle\in L}{\forall} \quad k = 1, ... , q \quad (8)$$

where $A(x)$ and $B(x)$ are sets of nodes as for the simple component flow, according to [11]. $f^k(x,y)$ is called the k-th component flow in the arc $<x,y>$. Let f(x,y) be a net flow in the arc $<x,y>$. If the arc $<x,y>$ is the oriented arc then the relation (9) applies.

$$f(x,y) = \sum_{k=1}^{q} f^k(x,y) \quad (9)$$

If the arc $<x,y>$ is unoriented, then the relation (10) applies.

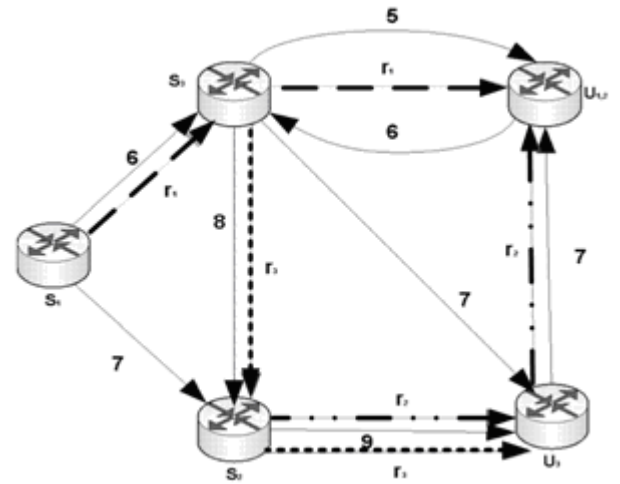Fig. 9. Multi-component flow

$$f(x,y) = \sum_{k=1}^{q} \left| f^k(x,y) - f^k(y,x) \right| \quad (10)$$

where $f^k(x,y)$ and $f(x,y)$ are, by definition, k-th component flows in oriented arcs $<x,y>$ and $<y,x>$ replacing the oriented arc $<x,y>$. The average packet delay is expressed by the relation (11) [11].

$$T = \frac{1}{\gamma} \sum_{<x,y>\in L} \frac{f(x,y)}{c(x,y) - f(x,y)} \quad (11)$$

where $\gamma = \sum_{i=1}^{q} r'_i$ is the net packet stream intensity injected into the networks from the outside. The relation (11) determines the average packet delay and has been formulated by Kleinrock in [13] with numerous assumptions associated to the computer network. The most important of these are:

- all network streams are stationary Poisson;
- all streams are static and independent;
- packet arrival instants are statistically independent of the their length;
- random packet lengths possess an exponential distribution;
- processing times at nodes are statistically independent;
- all network elements are fully reliable;
- each packet is sent to only single node;
- buffers capacities are infinite;

The authors of the paper have resolved that in sensor networks the delay introduced by the network is constant and the cost of information transmission is expressed in terms of the radiated power needed to deliver it. Therefore the authors have come to conclude that the most favorable solution is sending the data along such network (graph) paths which are characterized by the lowest propagation attenuation computed with a proper

propagation model. The purpose of this chapter was to manifest that the traditional methods for determining flow may not be germane to the current wireless mesh networks.

VIII. INFORMATION FLOW OPTIMIZATION

It is well known in the optimization theory [1]-[4] that the solution for which the shortest path is at the same time the least-hop path is the optimal solution for the wireless mesh networks. This statement is true because transmitting data along the shortest path with respect to the propagation attenuation means that the transmission has been performed using the least amount of energy in the transit nodes. The least-hop transmission, in turn, indicated that the packet transmission delay is reduced to the minimum.

Both these assertions (i.e. the least-hop transit and the shortest physical path selection) are satisfied by the proposed algorithm for the network topologies computed therewith. One can therefore design an optimal network in the sense of minimizing the maintenance and energy costs. In the case of sensor networks this feature will translate itself directly into maximization of the sensors battery lifetime.

As for wideband mobile networks, the data will be sent in an optimal fashion at a given instance thus minimizing the packet delay. The proposed algorithm of deploying mesh networks can be implemented as a topology-control function in order to assure the maximum number of nodes is served. As it has been aforementioned, the proposed solution has been thoroughly verified *via* a series of simulations. In fig. 11 an optimal path has been found with the proposed algorithm.

The route between nodes 3 and 13 is both the shortest (implying the lowest signal attenuation) and the least-hop one. For this reason the authors believe that the proposed topology assures that the shortest path between any pair of nodes *x* and *y* includes the least-hop path. It is easy to notice that there are 2 alternative routes between nodes 5 and 16, however only one of them is optimal with respect to the radio signal attenuation.
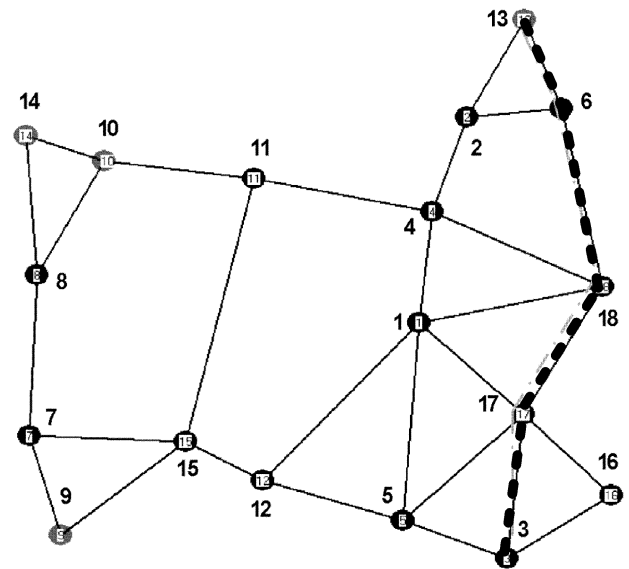

Fig. 11. Optimal path calculation

In the course of hundreds of simulations the authors have verified that in each case of the computed topology the shortest paths always included the least-hop path.

IX. CONCLUSIONS

The authors have proved that there exists an algorithm wherewith one can deploy a mesh network satisfying the theorem of dependability so that the graph branches do not cross (to avoid the interference issue). The time of execution is linear hence the algorithm can be applied as a topology control function for mobile mesh networks as well as for sensor networks. Simulations have been carried out for multiple scenarios yielding correct outcomes. The authors believe that the algorithm will be a valuable tool for designing and deploying mesh networks. Due to the analytic approach to the problem and algorithm of $O(n^3)$ computational complexity has been obtained which renders it suitable for implementation in users' terminals.

The authors have arrived to interesting observations in the course of multiple simulations as regards the planning of network topologies. These observations have been forged into principles that allow to solve the problem of minimum energy use in the case of battery-powered devices (such as sensors of mobile terminals) as well as the problem of minimizing the delay in stationary operator's networks. The outcomes illustrated herein with graphs can easily be implemented for ZigBee since its specification says that each ZigBee node may have up to 16 concurrent out- and incoming connections whereas in the simulations performed by the authors the topology control functions have always returned graphs with the number of enters and exits never exceeded 16.

In further research the authors intend to implement the topology control function for the ZigBee standard in order to verify whether a real physical system will indeed follow the mathematically computed communication topology. It is believed that the methods presented in this paper will allow one to fully exploit the network resources minimizing the delay in the network at the same time. Furthermore, the
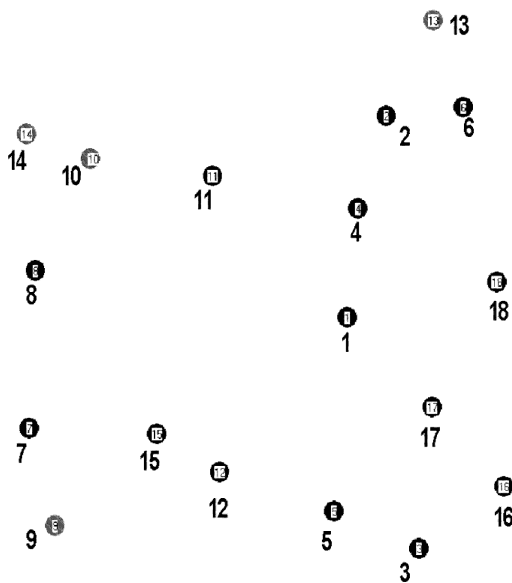

Fig. 10. A network deployment example

algorithm should eliminate intra-system interference and minimize the battery usage.

One should be mentioned that there exists some degree of similarity between the presented algorithm and solutions proposed earlier for satellite systems ([5]-[6]). In the case of a rosette constellation one should have properly connect satellites with the method of least triangles. However the solution space for satellite systems is finite since the satellites move around a sphere which a finite-area space. In the case of terrestrial mesh networks the solution space (a plane) is infinite. The authors have therefore formulated and solved the problem of three neighbors in order to assure that the mesh network is dependable (to the degree of two).

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