INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2010, VOL. 56, NO. 4, PP. 423–426
Manuscript received July 11, 2010; revised October 2010. DOI: 10.2478/v10177-010-0057-5

Loop Gain of the Common-Drain Colpitts Oscillator

Marian K. Kazimierczuk and Dakshina Murthy-Bellur

***Abstract*—This paper presents the derivations of the voltage transfer functions of the amplifier A, the feedback network β, and the loop gain T of the common-drain (CD) Colpitts oscillator, using the small-signal model of the CD Colpitts oscillator. The derivation of the characteristic equation of the CD Colpitts oscillator is presented. Using the characteristic equation, the equation for the oscillation frequency of the sinusoidal output voltage and the condition for steady-state oscillation are derived. The characteristic equation is used to obtain a plot of trajectories of the poles of the CD Colpitts oscillator by varying the MOSFET small-signal transconductance g_m. The locations of the complex conjugate poles depicting starting and steady-state conditions for oscillations are also presented.**

***Keywords*—Band-pass feedback networks, Colpitts oscillator, common-drain amplifier, location of poles, loop gain, oscillation conditions, positive feedback, resonant oscillators, tuned oscillators, voltage-controlled oscillators (VCO).**

I. Introduction

THE INTEGRATED CIRCUIT (IC) version of the single-transistor Colpitts oscillator [1]-[19] is widely used in radio frequency (RF) applications [5]-[9], [17]-[19]. Oscillators are also commonly used in various applications of power electronics [2], [3], [14]-[16]. The circuit of a common-drain oscillator is shown in Fig. 1. The derivation of the characteristic equation of the Colpitts oscillator using a BJT model is available in [1], [4], [7], and using the simplified MOSFET model is given in [5], [8], [19]. The derivation of the characteristic equation of the common-drain (CD) Colpitts oscillator using the small-signal model of the MOSFET including its output resistance r_o and the oscillator load resistance R_L, and the derivation of the loop gain of the CD Colpitts oscillator are not available in the literature. The objectives of this paper are (1) to derive the characteristic equation of the CD Colpitts oscillator, and (2) to derive the equation for the loop gain T of the CD Colpitts oscillator by deriving the amplifier gain A and the feedback network gain β, using the small-signal model. The characteristic equation is used to plot the trajectories of the poles of the CD Colpitts oscillator.

Section II presents the criteria for oscillations and the small-signal model of the CD Colpitts oscillator. The derivations of the characteristic equation and the loop gain are presented in Sections III and IV, respectively. Root locus plots of the CD Colpitts oscillator and conclusions follow in Sections V and VI, respectively.

M. K. Kazimierczuk is with the Department of Electrical Engineering, Wright State University, Dayton, OH, 45435 USA (e-mail: marian.kazimierczuk@wright.edu).

D. Murthy-Bellur is with the School of Engineering, The Pennsylvania State University, The Behrend College, Erie, PA 16563 USA (e-mail: murthybellur.2@wright.edu).

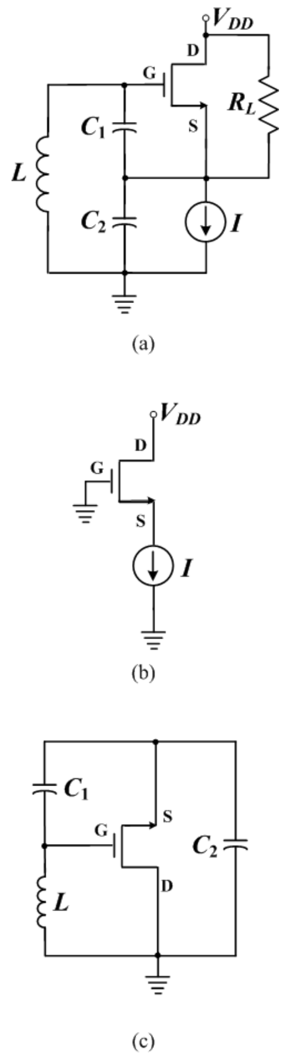

Fig. 1. Colpitts oscillator circuit with a transistor operating in common-drain (CD) configuration and biased by a current source I suitable for ICs; complete circuit (a), DC circuit (b), circuit for the ac component (c).

II. Single-Transistor Common-Drain Colpitts Oscillator

A. Criteria for Oscillations

An ac equivalent circuit of the single-transistor Colpitts oscillator in CD configuration is shown in Fig. 2. It consists of a noninverting amplifier whose open-loop voltage gain is denoted by A, and a noninverting frequency-selective feedback network whose voltage gain is denoted by β. The load resistance is denoted by R_L. The closed-loop gain A_f of the oscillator is given by

$$A_f = \frac{v_o}{v_f} = \frac{A}{1 - A\beta} = \frac{A}{1 - T}, \tag{1}$$

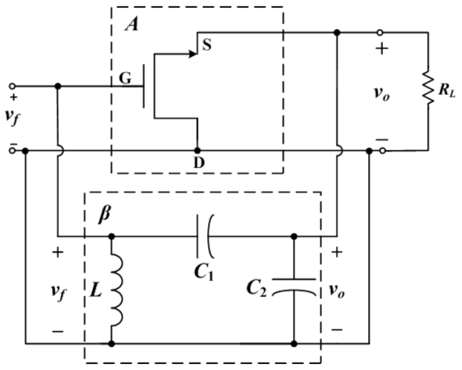

Fig. 2. A circuit of the single-transistor common-drain Colpitts oscillator for the ac component (biasing not shown).

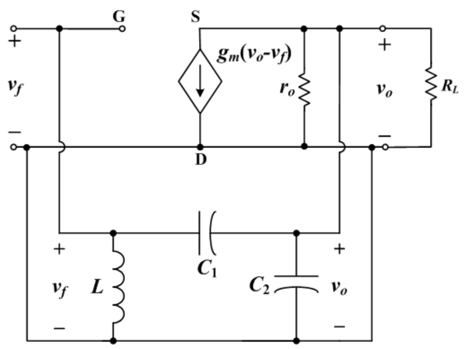

Fig. 3. Small-signal model of the single-transistor common-drain Colpitts oscillator.

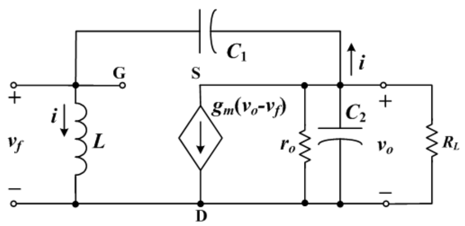

Fig. 4. Small-signal model of the single-transistor common-drain Colpitts oscillator with the feedback network shown together with the transistor model.

where $T = A\beta$ is the loop gain, v_o and v_f are the output and the feedback voltages, respectively. The condition for steady-state oscillation at the oscillation frequency f is given by

$$T(f) = A(f)\beta(f) = |T(f)|e^{j\phi_T(f)} = 1 + (0 \times j). \quad (2)$$

The Barkhausen magnitude criterion for steady-state oscillation at the oscillation frequency f is given by

$$|T(f)| = 1 \quad (3)$$

and the Barkhausen phase criterion for oscillation is

$$\phi_T(f) = 0 \pm n360^\circ. \quad (4)$$

At f, the magnitude of the loop gain must be equal to unity and the phase shift around the loop must be zero. The criterion of the real part of loop gain for oscillation is expressed as

$$Re[T(f)] = 1 \quad (5)$$

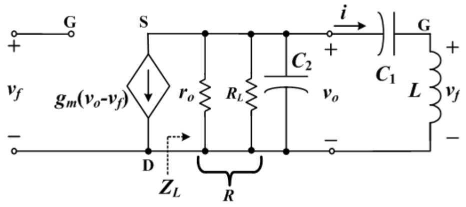

Fig. 5. Small-signal model of the common-drain Colpitts oscillator for determining the amplifier voltage gain A.

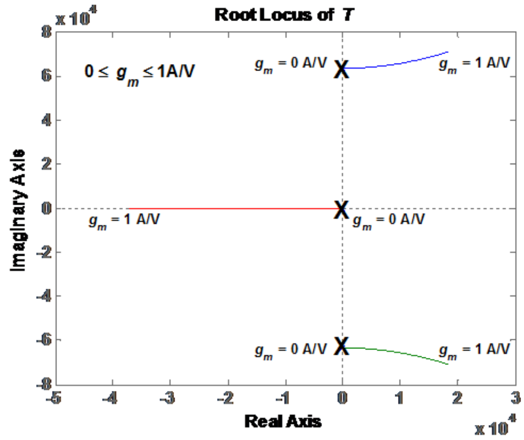

Fig. 6. Root locus plot of the closed-loop transfer function T of the common-drain Colpitts oscillator as g_m increases from 0 to 1 A/V.

and the criterion of the imaginary part of loop gain for oscillation is given by

$$Im[T(f)] = 0. \quad (6)$$

In order for the oscillations to start and grow, the magnitude of the loop gain T must be greater than unity.

B. Common-Drain Colpitts Oscillator Small-Signal Model

A small-signal model of the CD Colpitts oscillator is shown in Fig. 3. The MOSFET in Fig. 2 is replaced by a voltage dependent current source $g_m v_{gs}$ in parallel with the resistance r_o, where g_m is the small-signal transconductance of the MOSFET, r_o is the small-signal output resistance of the MOSFET, and $v_{gs} = v_f - v_o$. The MOSFET gate-to-source capacitance C_{gs} is included in the capacitance C_1. The MOSFET drain-to-source capacitance C_{ds} is included in the capacitance C_2. The MOSFET gate-to-drain capacitance C_{gd} is neglected. The output resistance of the transistor r_o and the load resistance R_L are included in R, where $R = R_L r_o/(R_L + r_o)$. The input resistance of the transistor is assumed to be very large.

III. CHARACTERISTIC EQUATION OF THE COMMON-DRAIN COLPITTS OSCILLATOR

For the sake of clarity, the feedback network shown in Fig. 3 is moved together with the amplifier model as shown in Fig. 4. Note that in Fig. 4, $v_{gs} = v_f - v_o$. The current through the series combination of L and C_1 is

$$i = \frac{v_o}{\left(sL + \frac{1}{sC_1}\right)} = v_o \left(\frac{sC_1}{1 + s^2 LC_1}\right), \quad (7)$$

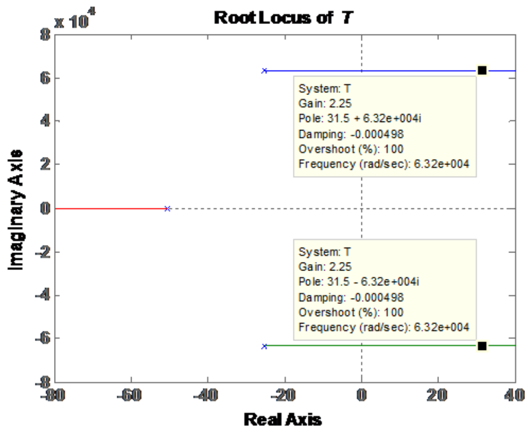

Fig. 7. Enlarged root locus plot depicting the complex conjugate poles in the right-half plane required to start oscillations.

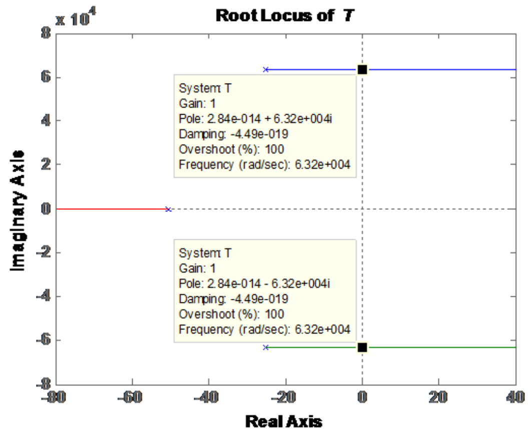

Fig. 8. Enlarged root locus plot depicting the complex conjugate poles on the imaginary axis required for the steady-state condition for oscillations.

and the current through L is

$$i = \frac{v_f}{sL}. \tag{8}$$

Equating (7) and (8), we have

$$v_o = v_f\left(\frac{s^2LC_1+1}{s^2LC_1}\right). \tag{9}$$

Applying KCL at the source terminal S in Fig. 4, we have

$$v_o\left(\frac{sC_1}{s^2LC_1+1}\right) + v_o\left(\frac{sC_2R+1}{R}\right) + g_m(v_o - v_f) = 0$$

$$v_o\left(\frac{sC_1}{s^2LC_1+1} + \frac{sC_2R+1}{R} + g_m v_o\right) - g_m v_f = 0. \tag{10}$$

Substituting (9) in (10), we have

$$v_f\left(\frac{s^2LC_1+1}{s^2LC_1}\right)\left(\frac{sC_1}{s^2LC_1+1} + \frac{sC_2R+1}{R} + g_m\right) - g_m v_f = 0. \tag{11}$$

Assuming the oscillations have begun, $v_f \neq 0$ in (11), therefore

$$\left(\frac{s^2LC_1+1}{s^2LC_1}\right)\left(\frac{sC_1}{s^2LC_1+1} + \frac{sC_2R+1}{R} + g_m\right) - g_m v_f = 0. \tag{12}$$

Simplifying (12) leads to

$$(LC_1C_2R)\,s^3 + (LC_1)\,s^2 + R\,(C_1+C_2)\,s + g_mR + 1 = 0. \tag{13}$$

Equation (13) is the characteristic equation of the CD Colpitts oscillator. Substituting $s = j\omega$ in (13) and rearranging the real and imaginary terms lead to

$$\left(1 + g_mR - \omega^2LC_1\right) + j\left[\omega R(C_1+C_2) - \omega^3LC_1C_2R\right]. \tag{14}$$

Equating the imaginary part of (14) to zero, the equation for the oscillation frequency is derived as

$$\omega_o = \frac{1}{\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}}\text{rad/s}, \tag{15}$$

or

$$f_o = \frac{1}{2\pi\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}}\text{Hz}. \tag{16}$$

Equating the real part of (14) to zero, and substituting (15), the condition for steady-state oscillations is derived as

$$g_mR = \frac{C_1}{C_2}. \tag{17}$$

Initially, for the oscillations to start and grow, the following inequality must be satisfied

$$g_mR > \frac{C_1}{C_2}. \tag{18}$$

IV. LOOP GAIN T OF THE COMMON-DRAIN COLPITTS OSCILLATOR

The loop gain transfer function T of the CD Colpitts oscillator is derived with the aid of small-signal model of the amplifier loaded by the feedback network shown in Fig. 5. Using the small-signal model of Fig. 5, the amplifier voltage gain A, and hence the loop gain $T = A\beta$ are obtained as follows. Using (9), the equation for the feedback gain β is found to be

$$\beta = \frac{v_f}{v_o} = \frac{s^2LC_1}{1+s^2LC_1}. \tag{19}$$

Referring to Fig. 5,

$$v_o = -g_m(v_o - v_f)Z_L \tag{20}$$

from which the amplifier voltage gain A is found as

$$A = \frac{v_o}{v_f} = \frac{-g_mZ_L}{s^2LC_1}, \tag{21}$$

where Z_L is the total load impedance seen by the amplifier given by

$$Z_L = \left(R||\frac{1}{sC_2}\right)||\left(sL + \frac{1}{sC_1}\right)$$
$$= \frac{R(1+s^2LC_1)}{s^3LC_1C_2R + s^2LC_1 + sR(C_1+C_2)+1}. \tag{22}$$

Substituting (22) in (21), the amplifier voltage gain A is found to be

$$A = -\frac{g_m}{s^2 LC_1}\left[\frac{R(1+s^2LC_1)}{s^3LC_1C_2R + s^2LC_1 + sR(C_1+C_2)+1}\right]. \quad (23)$$

By multiplying (19) and (23), the loop gain $T = A\beta$ is found as

$$T = A\beta = -\frac{g_m R}{s^3LC_1C_2R + s^2LC_1 + sR(C_1+C_2)+1}$$
$$= -\frac{\frac{g_m}{LC_1C_2}}{s^3 + s^2\frac{1}{C_2R} + s\frac{C_1+C_2}{LC_1C_2} + \frac{1}{LC_1C_2R}}. \quad (24)$$

The loop gain is described by a third-order voltage transfer function. It contains one real pole and two complex conjugate poles. For positive feedback, the feedback factor is $1-T=0$. Therefore, equating (24) to 1, we have

$$(LC_1C_2R)\,s^3 + (LC_1)\,s^2 + R\,(C_1+C_2)\,s + g_mR + 1 = 0. \quad (25)$$

Note that by utilizing the loop gain T, the characteristic equation obtained in (13) is derived alternatively as shown in (25).

V. ROOT LOCUS PLOTS OF THE CD COLPITTS OSCILLATOR

The characteristic equation of the CD Colpitts oscillator is given by (13). Dividing (13) by

$$(LC_1C_2R)\,s^3 + (LC_1)\,s^2 + R\,(C_1+C_2)\,s + 1, \quad (26)$$

we have

$$1 + \frac{g_mR}{(LC_1C_2R)\,s^3 + (LC_1)\,s^2 + R\,(C_1+C_2)\,s + 1} = 0. \quad (27)$$

Using (27), the root locus plots of the CD Colpitts oscillator with g_m as a variable and $L = 50$ μH, $C_1 = C_2 = 10$ μF, r_o = 100 kΩ, R_L = 1 kΩ, and R = 990 Ω are shown in Figs. 6 - 8. As g_m increases from 0 to 1 A/V, the real pole moves to the left and the two complex poles move from left-half plane (LHP) to the right-half plane (RHP) as shown in Fig. 6. For the oscillations to start and grow, the two complex conjugate poles must be in RHP as shown in Fig. 7 satisfying the condition given in (18). For steady-state oscillations, the two complex poles must be located on the imaginary axis as shown in Fig. 8. From Fig. 8, it can be seen that when the complex conjugate poles are on the imaginary axis, the values of gain $g_mR = 1$ and frequency $\omega_o = 63.2$ krad/s are in good agreement with the theoretical values calculated from equations (17) and (15), respectively.

VI. CONCLUSIONS

This paper has presented the derivations of the loop gain T and the characteristic equation of the single-transistor common-drain Colpitts oscillator, using the small-signal model. The equation for the loop gain T has been derived by first deriving the amplifier voltage gain A and the feedback network gain β. The loop gain T is a third-order voltage transfer function containing one real pole and two complex conjugate poles. The equation for oscillation frequency and the condition for steady-state oscillation have been derived using the characteristic equation. By varying the MOSFET transconductance g_m, a plot of the trajectories of the poles of the CD Colpitts oscillator has been given. The equation derived for the steady-state oscillation condition has been verified by investigating the locations of complex conjugate poles at the frequency of oscillation. In order for the output voltage oscillations to start and grow to a steady-state level, the location of the complex conjugate poles must lie in the RHP and move towards the LHP as g_m decreases, and finally lie on the imaginary axis for marginal stability. Analysis of the common-drain Colpitts oscillator with gate-to-drain capacitance C_{gd} is suggested for future work.

REFERENCES

[1] K. K. Clarke and D. T. Hess, *Communication Circuits: Analysis and Design*. Reading, MA. Addison-Wesley, 1971.

[2] J. Ebert and M. K. Kazimierczuk, "Class E high-efficiency tuned power oscillator," *IEEE Journal of Solid State Circuits*, vol. SC-16, no. 2, pp. 62–66, Apr. 1981.

[3] M. K. Kazimierczuk, "A new approach to the design of tuned power oscillators," *IEEE Transactions on Circuits and Systems*, vol. CAS-29, pp. 261–267, Apr. 1982.

[4] A. S. Sedra and K. C. Smith, *Microelectronic Circuits*. New York. Oxford University Press, 1991.

[5] Q. Huang, "Power consumption vs LO amplitude for CMOS Colpitts oscillators," in *IEEE Custom Integrated Circuits Conference*, Santa Clara, CA, May11-16 1997, pp. 255–258.

[6] G. M. Maggio, O. D. Feo, and M. P. Kennedy, "Nonlinear analysis of the Colpitts oscillator and applications to design," *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, vol. 46, no. 9, pp. 1118–1129, Sept. 1999.

[7] G. M. Maggio and M. P. Kennedy, "Classification of the steady-state behaviour of the Colpitts oscillator," in *IEEE International Conference on Electronics, Circuits and Systems (ICECS)*, Pafos , Cyprus, Sept.11-16 1999, pp. 811–814.

[8] K. Mayaram, "Output voltage analysis for the MOS Colpitts oscillator," *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, vol. 47, no. 2, pp. 260–263, Feb. 2000.

[9] A. Tamasevicius, G. Mykolaitis, S. Bumeliene, A. Cenys, A. Anagnostopoulos, and E. Lindberg, "Two-stage chaotic Colpitts oscillator," *Electronics Letters*, vol. 37, no. 9, pp. 549–551, Apr. 2001.

[10] R. R. Spencer and M. S. Ghausi, *Introduction to Electronic Circuit Design*. Upper Saddle River, NJ. Prentice-Hall, 2003.

[11] R. C. Jaeger and T. N. Blalock, *Microelectronic Circuit Design*. New York. McGraw-Hill, 2004.

[12] A. Aminian and M. K. Kazimierczuk, *Electronic Devices: A Design Approach*. Upper Saddle River, NJ. Prentice-Hall, 2004.

[13] T. H. Lee, *The Design of Radio Frequency Integrated Circuits*. New York. Cambridge University Press, 2004.

[14] M. K. Kazimierczuk, V. G. Krizhanovski, J. V. Rassokhina, and D. V. Chernov, "Class-E MOSFET tuned power oscillator design procedure," *IEEE Transactions on Circuits and Systems-I: Regular Papers*, vol. 52, no. 6, pp. 1138–1147, June 2005.

[15] ——, "Injection-locked class-E oscillator," *IEEE Transactions on Circuits and Systems-I: Regular Papers*, vol. 53, no. 6, pp. 1214–1222, June 2006.

[16] V. G. Krizhanovski, D. V. Chernov, and M. K. Kazimierczuk, "Low-voltage electronic ballast based on class E oscillator," *IEEE Transactions on Power Electronics*, vol. 22, no. 3, pp. 863–870, May 2007.

[17] I. M. Filanovsky, C. J. M. Verhoeven, and M. Reja, "Remarks on analysis, design and amplitude stability of MOS Colpitts oscillator," *IEEE Transactions on Circuits and Systems-II: Express Briefs*, vol. 54, no. 9, pp. 800–804, Sept. 2007.

[18] R. Devine and M.-R. Tofighi, "Class E Colpitts oscillator for low power wireless applications," *Electronics Letters*, vol. 44, no. 21, pp. 549–551, Oct. 2008.

[19] A. S. Elwakil, "On the two-port network classification of Colpitts oscillators," *IET Circuits, Devices and Systems*, vol. 3, no. 5, pp. 223–232, May 2009.