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Some Principles of Network Calculus Revisited

Andrzej Borys, Mariusz Aleksiewicz, Dariusz Rybarczyk, and Katarzyna Wasielewska

***Abstract*—Network calculus is a mathematical theory dealing with queueing problems in packet-switched computer networks. It provides algorithms to determine resource requirements of traffic flows using arrival and service curves and describes delays and backlogs in network systems. Network calculus framework is based on a min-plus algebra which allows to transform complex network optimization problems into analytically tractable ones. Recently, a fundamental book on principles, tools, techniques, and applications of network calculus, entitled: Network Calculus. A Theory of Deterministic Queuing Systems for the Internet, has been published by J. Y. Le Boudec and P. Thiran. Here, we refer to it in our refinements of proof of one important theorem and its extension. The objective of this paper is twofold. First, we complete one of basic results regarding a network element that is called in network calculus a greedy shaper. Second, we present also the results of some illustrative calculations and measurements of network service curve. They aim in better understanding of its properties.**

***Keywords*—Greedy shapers, network calculus, service curve.**

I. Introduction

NETWORK calculus is a system theory for queueing systems which is based on min-plus algebra. This technique is used for analysis and optimization of flow control in computer networks. Network calculus analysis focuses on obtaining bounds for performance metrics instead of an exact traffic and service characterization. There are deterministic and stochastic approaches to network calculus. This technique is used for bandwidth estimation in both wired and wireless networks.

First experiences with network calculus can be found in work of Cruz [1]. In his seminal papers [1], the difficulty of bounding end-to-end delay in computer networks is discussed and the use of so-called regulators to increase the throughput is proposed. Extended description of network calculus theory can be found in [2]. In our paper, we give more precise explanation of the greedy shaper definition and we present a detailed proof of a basic theorem regarding this network element. Additionally, we illustrate the service curve properties.

Some results of this paper were presented by the authors at the conference [3], [4]. It is organized as follows. Section II presents the main network calculus operations. In Section III, the role of the traffic causality property in considerations leading to formulation of the notion of a greedy shaper is explained in detail. Furthermore, the necessary and sufficient conditions for a shaper to be a greedy shaper are given in a form of theorem. The next section is devoted to the illustration of service curve properties. And finally, the last section contains summary and conclusions.

A. Borys and M. Aleksiewicz are with the Institute of Telecommunications, Faculty of Telecommunications and Electrical Engineering, University of Technology and Life Science, Bydgoszcz, Poland (e-mails: {andrzej.borys, mariusz.aleksiewicz} @utp.edu.pl).

D. Rybarczyk is with the OPEN SYSTEM Dariusz Rybarczyk, Bydgoszcz, Poland (e-mail: dariusz.rybarczyk@hotmail.com).

K. Wasielewska is with the Institute of Applied Informatics, The State School of Higher Professional Education in Elblag, Poland (e-mail: k.wasielewska@gmail.com).

II. Network Calculus

Network calculus introduces concepts that can be used to characterize elements in communication networks. It describes cumulative functions for input and output data flows and is useful to calculate network delays, backlogs and other Quality of Service parameters. We consider a data flow passing through a network. In network calculus a node behaviour is characterized by a function called the *service curve* which denotes how long a packet must be serviced after an arrival to a node. The input traffic is characterized by a wide-sense increasing function of time and it is so-called the *arrival curve*. This function quantifies constraints on the number of bits of packet flow in the time interval at service node.

A. Some Definitions

Let $\mathcal{F}$ be a class of real-valued and wide-sense increasing functions for $t \geq 0$ with t meaning a time variable and having values identically for $t < 0$ equal to zero. In particular, let $R(t), S(t) \in \mathcal{F}$ be cumulative input and output flows, respectively. The cumulative traffic stands here for the sum of bits (packets) arrived at the input or output in the period from 0 to t.

For given functions f and g network calculus defines a *convolution* operator $\otimes$ by

$$(f \otimes g)(t) = \inf_{0 \leq \tau \leq t} \{f(\tau) + g(t-\tau)\},$$

and a *deconvolution* operator $\oslash$ by

$$(f \oslash g)(t) = \sup_{\tau \geq 0} \{f(t+\tau) - g(\tau)\}.$$

In network calculus, $R(t)$ can be upper bounded by an arrival curve. It means that for all $0 \leq s \leq t$ we have

$$R(t) - R(s) \leq \alpha(t-s),$$

where α is called the *arrival curve* of R. Equivalently, for all $t \geq 0$ we have

$$R(t) \leq (R \otimes \alpha)(t).$$

We also say that R is α*-smooth*. We say that service node offers a *service curve* β to the flow if and only if for all $t \geq 0$ there exists some s such that

$$S(t) - R(s) \geq \beta(t-s).$$

Equivalently, for all the input functions $R(t)$ and its corresponding output functions $S(t)$ we have

$$S(t) \geq (R \otimes \beta)(t).$$

A function $f \in \mathcal{F}$ is *sub-additive* iff for all $s, t \geq 0$

$$f(s+t) \leq f(s) + f(t).$$

If flow has arrival curve $f \in \mathcal{F}$ then it also has sub-additive arrival curve

$$\bar{f}(x) \leq f(x) \text{ for all } x.$$

Let $f \in \mathcal{F}$. If $f(0) = 0$ then $f \geq f \otimes f \geq 0$. By repeating this operation we will obtain a sequence of functions that are every time smaller and which converges to some function that is limited and the largest sub-additive function smaller than f and having zero in $t = 0$. This function is called *sub-additive closure* of f. More formal definition is as follows. Let f will be a function or sequence of $\mathcal{F}$. Denote $f^{(n)}$ the function obtained by repeating $(n-1)$ convolutions of f with itself, that is $f^{(0)} = \delta_0$ (where δ_0 is a fixed function defined by $\delta_0(t) = \infty$ for $t > 0$ and $\delta_0(0) = 0$), $f^{(1)} = f$, $f^{(2)} = f \otimes f$, etc. Then the sub-additive closure of f, denoted by $\bar{f}$, is defined by

$$\bar{f} = \inf_{n \geq 0} \left\{ f^{(n)} \right\}.$$

A function f is named 'good' function if anyone of the following equivalent properties is satisfied [2]:

1) f is sub-additive and $f(0) = 0$,
2) $f = f \otimes f$,
3) $f \oslash f = f$,
4) $f = \bar{f}$.

In [2] it has been shown that the sub-additive closure of a function f is the largest 'good' function $\bar{f}$ such that $\bar{f} \leq f$ and that $\bar{f} \in \mathcal{F}$ if $f \in \mathcal{F}$.

B. Input-Output Characterization of Greedy Shapers

If we want to ensure in a network some QoS guarantees, we must, first of all, constrain its input flows. This is done by arrival curves. This condition can be achieved by shaping the input traffic with *shaping curve* σ. A *shaper* forces that the packets which are entered to the queue are constrained by an arrival curve σ. A *greedy shaper* is a shaper that shapes a flow of an input bits in a buffer with the constraint σ and outputs them as soon as possible. In [2] we can find important theorem associated with the greedy shapers (Theorem 1.5.1 in [2]):

Theorem 1: Consider a greedy shaper with shaping curve σ. Assume that the shaper buffer is empty at time 0, and that it is large enough so that there is no data loss. For an input flow R, the output S is given by

$$S = R \otimes \bar{\sigma},$$

where $\bar{\sigma}$ is the sub-additive closure of σ.

III. Refinement of Greedy Shaper Definition

In the proof of theorem 1.5.1 in [2], a virtual system satisfying the conditions of traffic causality and $\bar{\sigma}$-smoothness is considered. That is a traffic system of which input $R(t)$ and output $S(t)$ cumulative traffics, where t is a time variable, fulfill the following set of inequalities

$$\begin{cases} S(t) \leq R(t) & \text{(1a)} \\ S(t) \leq (S \otimes \bar{\sigma})(t) & \text{(1b)} \end{cases}$$

for each value of $t \geq 0$. Moreover, it is assumed that the functions $S(t)$, $R(t)$, and $\bar{\sigma}(t)$ belong to the so-called $\mathcal{F}$ class of functions. The function $\bar{\sigma}$ in (1b) is assumed to be a "good" function [2] that is satisfying the sub-additivity property [2] and having the value zero for $t = 0$ ($\bar{\sigma}(0) = 0$).

The interpretation of inequalities (1a) and (1b) is as follows. Consider first (1a). It expresses the constraint imposed by traffic causality on the virtual system considered, saying that the bits transmitted in it can not appear at its output earlier than at its input. Further, the second condition, expressed by (1b), says that the system output traffic $S(t)$ is constrained by an arrival curve $\sigma(t)$, being a function belonging to $\mathcal{F}$ and of which sub-additive closure is denoted here as $\bar{\sigma}(t)$. That is, according to the definition of an arrival curve [2], we have

$$S(t) - S(s) \leq \sigma(t-s). \tag{2}$$

Moreover, note that (2) can be equivalently rewritten as

$$S(t) \leq (S \otimes \sigma)(t). \tag{3}$$

Furthermore, it has been shown in [2] that σ in (3) can be always replaced by its sub-additive closure, that is by $\bar{\sigma}$. This follows from the result proved that if σ is a system arrival curve then its sub-additive closure $\bar{\sigma}$ is, too. In what follows, we use (3) with $\bar{\sigma}$ instead of σ, that is

$$S(t) \leq (S \otimes \bar{\sigma})(t). \tag{4}$$

And finally, observe that (4) is exactly the same inequality as (1b).

The theorem 1.5.1 proved in [2] is fundamental for the theory of greedy shapers. It says that the output traffic $R^*(t)$ of a greedy shaper is equal to $(R \otimes \bar{\sigma})(t)$, where $R(t)$ represents its input traffic. That is every greedy shaper possesses the input-output representation in the form

$$R^*(t) = (R \otimes \bar{\sigma})(t). \tag{5}$$

Denote now $S^* = R \otimes \bar{\sigma}$. In [2] (in proof of lemma 1.5.1), it was only mentioned that since $\bar{\sigma}$ is a "good" function S^* is a solution of the set of inequalities (1). We prove this statement here in detail; it is not obvious. To this end, we introduce S^* in (1b) in place of S. It means that

$$R \otimes \bar{\sigma} \leq (R \otimes \bar{\sigma}) \otimes \bar{\sigma} = R \otimes (\bar{\sigma} \otimes \bar{\sigma}) = R \otimes \bar{\sigma}. \tag{6}$$

So S^* is really a solution in (1b). Note also that the associativity of the $\otimes$ operation and the fact that $\bar{\sigma} \otimes \bar{\sigma} = \bar{\sigma}$ [2] were exploited on the right-hand side of (6).

A little bit more difficult task is to prove that S^* is also a solution of (1a). To begin, let us rewrite $S^* = R \otimes \bar{\sigma}$ as

$$S^*(t) = (R \otimes \bar{\sigma})(t) = \inf_{0 \leq s \leq t} \{R(s) + \bar{\sigma}(t-s)\}. \tag{7}$$

Then, note that the expression of which infimum is calculated in (7) equals

$$R(t) + \bar{\sigma}(0) = R(t) + 0 = R(t) \tag{8}$$

for $s = t$. In (8) we have applied the property of the sub-additive closure of a function of possessing the zero value for $t = 0$. So if the infimum of (7) would occur for $s = t$ then the inequality (1b) would be satisfied because we would have $R(t) \leq R(t)$. Furthermore, if the infimum in (7) would occur for some other s, different from t, it had to be equal or less than $R(t)$. The reverse inequality

$$\inf_{0 \leq s \leq t} \{R(s) + \bar{\sigma}(t-s)\} > R(t) \quad (9)$$

would contradict the notion of infimum operation. So, concluding, we really have

$$S^* = R \otimes \bar{\sigma} \leq R, \quad (10)$$

and this means that S^* satisfies also (1a).

In the lemma 1.5.1 underlying the proof of theorem 1.5.1 in [2], it is shown that the solution $S^* = R \otimes \bar{\sigma}$ is maximal. That is any other solution, say $S^{'}$, must satisfy the following inequality

$$S^{'} \leq S^* = R \otimes \bar{\sigma}. \quad (11)$$

The proof of (11) given in [2] needs also some refinement. We present it in what follows below, starting with

$$S^{'} \leq R, \quad (12)$$

which is true because, after our assumption, $S^{'}$ is the solution of the system of inequalities (1), in particular of (1a).

Adding $\bar{\sigma}(t-s)$ on both sides of (12) gives

$$S^{'}(s) + \bar{\sigma}(t-s) \leq R(s) + \bar{\sigma}(t-s) \quad (13)$$

for all s and t. Therefore, (13) will be also true when the infimum operation is applied on the right-hand side of (13). That is

$$S^{'}(s_i) + \bar{\sigma}(t-s_i) \leq (R \otimes \bar{\sigma})(t) \quad (14)$$

holds, where s_i means the value of s for which the infimum of $R(s) + \bar{\sigma}(t-s)$ occurs. Obviously, according to the definition of infimum operation, we have

$$(S^{'} \otimes \bar{\sigma})(t) \leq S^{'}(s_i) + \bar{\sigma}(t-s_i). \quad (15)$$

Applying the result (15) in (14) gives finally

$$S^{'} \otimes \bar{\sigma} \leq R \otimes \bar{\sigma}. \quad (16)$$

Now we recall the fact that $S^{'}$ satisfies also (1b). That is we can write

$$S^{'} \leq S^{'} \otimes \bar{\sigma}. \quad (17)$$

Using then (17) in (16), we get

$$S^{'} \leq R \otimes \bar{\sigma} = S^*. \quad (18)$$

and (18) is identical with (11). That is any other solution of (1) different from S^* satisfies (11).

We remark now that the fact of having by the greedy shaper the input-output representation in the form given by (5) can be also used in formulation of an equivalent definition of the greedy shaper. So, we can define it as such a shaper of which output response is the maximal solution to the set of inequalities (1) determining its behaviour. In other words, this can be formulated in a more formal form by the following theorem.

Theorem 2: A traffic shaper is a greedy shaper if and only if its output traffic R^* is related to its input traffic by

$$R^* = R \otimes \bar{\sigma},$$

with $\bar{\sigma}$ meaning the sub-additive closure function of its shaping curve σ.

Proof: We start with the necessary condition first, and remark that, in fact, the theorem 1.5.1 in [2] is its proof. Really, if this was not the case, we would have $R^*(t) \neq S^*(t)$. Let us now check whether it would be possible. Certainly, $R^*(t) \leq S^*(t)$ would be satisfied because $R^*(t)$ had to fulfill the set of inequalities (1) as a shaper. However, the assumption $R^*(t) \neq S^*(t)$ for all or for some times t could not hold because then this shaper would not release bits from the buffer as early as possible. So, therefore, it would not be a greedy shaper.

Now we prove that $R^* = R \otimes \bar{\sigma}$ is the sufficient condition of being a greedy shaper. To this end, observe first that S^* is maximal, that is the best solution for a traffic shaper described by (1). And whence, it describes behaviour of a shaper that sends out bits from its buffer as early as only possible. So it describes a greedy shaper, and this ends the proof. ■

It is very important for researchers to understand all theoretical basis of network calculus. In [3] Borys et al. have discussed and explained another complicated terms and statements in the area of the greedy shapers and some traffic metrics.

IV. Service Curve Illustration

This section illustrates properties of the service curve in the time domain on two examples of simple teletraffic networks, possessing peak-rate or rate-latency service curve. We are presenting detailed calculations of a network output traffic for different rates of service curve, and then we are summarizing the results in figures. All the main outcomes are verified experimentally.

A. Systems Possessing Peak-Rate Type Service Curve

Consider a linear time-invariant causal teletraffic system (network). Such a system has an input-output representation in the form of so-called infimum convolution

$$y(t) = (\beta \otimes x)(t) = \inf_{0 \leq \tau \leq t} \{\beta(\tau) + x(t-\tau)\},$$
$$\beta(\tau) \equiv 0 \text{ for } \tau < 0 \quad (19)$$

where inf means the mathematical operation of taking infimum value. Moreover, $y(t)$ and $x(t)$ in (19), being the functions of time t, are the cumulative output and input traffics, respectively, in the system. The cumulative input or output traffic stands here for the sum of bits (packets) arrived at the input or output, respectively, in the period from 0 to t. The function $\beta(\tau)$ is a service curve of a given teletraffic system.

The simplest service curve is the peak-rate function [2] given by

$$\beta(t) = \begin{cases} rt & \text{if } t > 0 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where r is a constant (independent of time) and means the rate.

Consider now a system possessing the service curve of the form (20) with the input traffic applied to it described by a similar function, that is

$$x(t) = \begin{cases} Rt & \text{if } t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where R means the rate of the transmitted traffic. We shall consider the following values of the rate r in $\beta(t)$ given by (20)

$$r = r_1 < R \Rightarrow (r_1 - R) < 0, \tag{22a}$$

$$r = r_2 = R \Rightarrow (r_2 - R) = 0, \tag{22b}$$

$$r = r_3 > R \Rightarrow (r_3 - R) > 0. \tag{22c}$$

Moreover, we denote here β_1 for r_1, β_2 for r_2, and β_3 for r_3, accordingly. Using now (20-22c) in (19) for i =1,2,3 we get

$$y(t) = \inf_{0 \le \tau \le t} \{(r_i - R)\tau + Rt\}. \tag{23}$$

Observe that Rt in the inner expression in the operation inf in (23) is constant for a given time t. Taking into account the results given on the right-hand sides of (22a-22c), we obtain

$$y(t) = \begin{cases} y_1(t) = r_1 t & \text{(24a)} \\ y_2(t) = Rt = r_2 t & \text{(24b)} \\ y_3(t) = Rt & \text{(24c)} \end{cases}$$

The system output traffic given by (24a-24c) is visualized in Fig. 1.

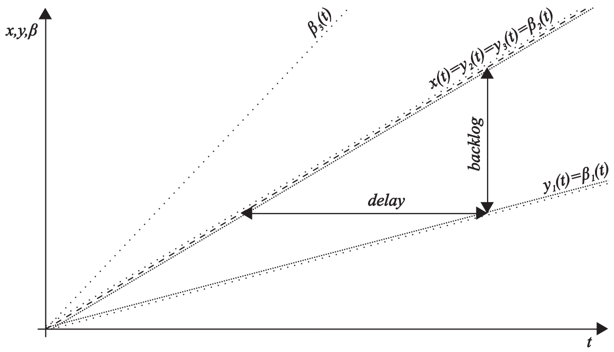

Fig. 1. Illustration of the output traffic of a teletraffic system possessing a peak-rate type service curve for three characteristic values of the rate r_i.

Observe from Fig. 1 that the whole input traffic goes through a system in the cases 2 and 3 without any delay and backlog. In opposite to this, in the case 1, the too small value of r_1 causes the delay and backlog of the transmitted data increasing to infinity.

B. Systems Possessing Rate-Latency Type Service Curve

The second simple and most typical service curve occurring in teletraffic systems is the rate-latency function [2] given by

$$\beta_T(t) = r[t - T]^+ = \begin{cases} r(t - T) & \text{if } t > T \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

where r means the rate and T the delay (both independent of time). Moreover, the notation $[x]^+$ denotes max of $\{x, 0\}$.

Consider now a system possessing the service curve of the form (25) with the input traffic applied to it which is given, similarly as before, by a function (21). We shall consider also in the case of the rate-latency service curve the three characteristic values of the rate r given by (22a-22c). For these values, we shall denote β_{T1} for r_1, β_{T2} for r_2, β_{T3} for r_3, accordingly.

Introducing (21) and (25) into (19) gives

$$y(t) = \inf_{0 \le \tau \le t} \big\{ r_i[\tau - T]^+ + R(t - \tau) \big\}. \tag{26}$$

The infimum convolution is cumulative [2], therefore, we can rewrite (26) in an equivalent form as

$$y(t) = \inf_{0 \le \tau \le t} \big\{ r_i[t - T - \tau]^+ + R\tau \big\}. \tag{27}$$

In our further analysis, we shall use the formula (27) and consider in it the times $t \le T$ first. This allows us to write $t - T - \tau \le 0$ for $\tau \ge 0$, what applied in (27) gives

$$y(t) = \inf_{0 \le \tau \le t} \{0 + R\tau\} = R \cdot 0 = 0. \tag{28}$$

Next we consider the times $t > T$, that are the ones for which $t - T > 0$ holds. We must consider two cases: $t - T - \tau \le 0$ and $t - T - \tau > 0$. In the first case, we get the possible change of τ restricted to $t - T \le \tau \le t$. In the second case, we obtain the change of τ restricted to $0 \le \tau \le t - T$. Further, note that the sum of these sets is identical with the set $\{\tau : 0 \le \tau \le t\}$, that is $\{\tau : 0 \le \tau \le t - T\} \cup \{\tau : t - T \le \tau \le t\} = \{\tau : 0 \le \tau \le t\}$. This allows us to apply, in what follows, the following formula

$$\inf_{S = S_1 \cup S_2} \{\cdot\} = \inf_2 \Big\{ \inf_{S_1} \{\cdot\}, \inf_{S_2} \{\cdot\} \Big\}, \tag{29}$$

where the set S is a sum of subsets S_1 and S_2, and number 2 under the second inf symbol in (29) means taking infimum of a set consisting of two elements.

Consider now the case (22a) in (27) for times $t > T$. We get then

$$\begin{aligned} &\inf_{0 \le \tau < t - T} \{r_1(t - T - \tau) + R\tau\} = \\ &\inf_{0 \le \tau < t - T} \{(R - r_1)\tau + r_1(t - T)\} = r_1(t - T) \end{aligned} \tag{30}$$

because $R - r_1 > 0$, and

$$\inf_{t - T \le \tau \le t} \{0 + R\tau\} = R(t - T). \tag{31}$$

Applying next (29) with the results (30) and (31) gives

$$\begin{aligned} y_1(t) = &\inf_{0 \le \tau \le t} \big\{ r_1[t - T - \tau]^+ + R\tau \big\} = \\ &\inf_2 \{r_1(t - T), R(t - T)\} = r_1(t - T) \end{aligned} \tag{32}$$

because $r_1 < R$.

Consider now the case (22b) in (27) for the times $t > T$. We get then

$$\begin{aligned} &\inf_{0 \le \tau < t - T} \{r_2(t - T - \tau) + R\tau\} = \\ &\inf_{0 \le \tau < t - T} \{0 \cdot \tau + r_2(t - T)\} = r_2(t - T) \end{aligned} \tag{33}$$

because $R - r_2 = 0$, and

$$\inf_{t-T\le\tau\le t}\{0 + R\tau\} = R(t-T). \tag{34}$$

Applying next (29) with the results (33) and (34) gives

$$\begin{aligned} y_2(t) &= \inf_{0\le\tau<t}\left\{r_2[t-T-\tau]^+ + R\tau\right\} = \\ &\inf_2\{r_2(t-T), R(t-T)\} = r_2(t-T) = R(t-T) \end{aligned} \tag{35}$$

because $r_2 = R$.

Consider now the case (22c) in (27) for the times $t > T$. We get then

$$\begin{aligned} &\inf_{0\le\tau<t-T}\{r_3(t-T-\tau) + R\tau\} = \\ &\inf_{0\le\tau<t-T}\{r_3(t-T) + (R-r_3)\tau\} = R(t-T) \end{aligned} \tag{36}$$

because $R - r_3 < 0$, and

$$\inf_{t-T\le\tau\le t}\{0 + R\tau\} = R(t-T). \tag{37}$$

Applying next (29) with the results (36) and (37) gives

$$\begin{aligned} y_3(t) &= \inf_{0\le\tau\le t}\left\{r_3[t-T-\tau]^+ + R\tau\right\} = \\ &\inf_2\{R(t-T), R(t-T)\} = R(t-T). \end{aligned} \tag{38}$$

The system output traffic given by (28) with (32), or with (35), or with (38) is visualized in Fig. 2.

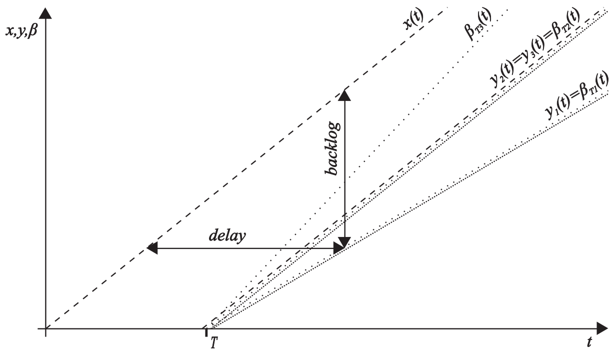

Fig. 2. Illustration of the output traffic of a teletraffic system possessing a rate-latency type service curve for three characteristic values of the rate r_i.

Observe from Fig. 2 that the delay and backlog are constant, that is they do not change with time, in the cases 2 and 3. In opposite to this, in the case 1, as before, the too small value of r_1 causes the increase of the delay and backlog of the transmitted data going to infinity with the time increase.

C. Measurement-Based Service Curve Estimation Scheme Exploiting Deconvolution

The deconvolution operation for input and output functions, defined as

$$(y \oslash x)(t) = \sup_{\tau\ge0}\{y(t+\tau) - x(\tau)\}, \tag{39}$$

can be used in a measurement-based estimation of the system service curve. More precisely, it has been proved in [5] that for linear, causal, and time-invariant teletraffic systems the calculated function

$$\tilde\beta(t) = \sup_{\tau\ge0}\{y(t+\tau) - x(\tau)\} \tag{40}$$

is a lower bound of the service curve $\beta(t)$ of the system for which $y(t) = (\beta \otimes x)(t)$ holds. That is for this system, we have

$$\tilde\beta(t) \le \beta(t). \tag{41}$$

Moreover, it has been also shown in [5] that $\beta(t)$ given by (40) with $y(t+\tau)$ and $x(t)$ representing the measured values of the cumulative output and input traffic of the system for the corresponding times in the best possible estimate of $\beta(t)$ which can be obtained from measurements.

The method of estimation of the service curve as outlined above, the calculation scheme and estimation algorithm using (40) have been chosen in this paper.

Choosing the estimation method described shortly above, we were aware that it was not ideal, and has some disadvantages. Some of them have been already reported in [5], however, one important has been omitted. It regards the following: there are times t, ranging from 0 to some t_0, for which $y(t+\tau) < x(\tau)$ for all τ's, giving $\sup_{\tau\ge0}\{\cdot\} < 0$. Because the service curve must fulfill $\beta(t) \ge 0$, it is reasonable to require that $\tilde\beta(t)$ does not assume negative values. So, in the algorithm, every time, when we get $\sup_{\tau\ge0}\{\cdot\} < 0$, we should set $\tilde\beta(t) = 0$. We must remember this.

D. Practical Verification

To verify our observations made with regard to the service curves of the peak-rate and rate-latency types, which enable better understanding the notion of the service curve used for teletraffic systems, we carried out measurements of the input and output traffic in a measuring setup shown in Fig. 3.

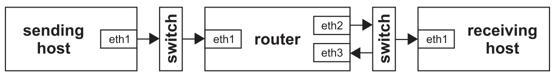

Fig. 3. A scheme of a measuring setup used.

The measuring setup of Fig. 3 works in the following way: the sending host is connected to the router via the switch working in accordance with the standard 1000base-T, and sends the datagrams UDP. The router registers the arrival times of datagrams (interface eth1), and afterwards sends them to the receiving host through the interface eth2. Switch on the right-hand side of Fig. 3, working in accordance with the forced standard 10base-T. The data sent by this switch to the receiving host are sent at the same time to the interface eth3 too. In the interface eth3, the times of their arrivals are registered, and they are identical with the arrivals times of the above data at the interface eth1 at the receiving host. So, in this time measuring scheme used here, both the arrival and departure times are measured on the same system element (router). Thereby, we avoid the possible errors which would occur as a result of lack of time synchronization between the different hosts.

The data have been sent in the measuring setup of Fig. 3 in two series:

1) with the rate 1050 dat(datagrams)/s, each datagram consisting of 1120 B (what gives 9392 bits, inclusive 96 bits of interframe period), in consequence giving the following rate in bit/s: 9861600 bit/s;
2) with the rate twice of that from point 1, that is 19723200 bit/s.

Note that in point 1 above the data rate was a little bit less than the maximal rate allowed by connection router eth2-switch-eth1 of the receiving host in Fig. 3 equal to 10 Mbit/s. But, in point 2, the data rate was greater than the maximal rate allowed by the connection router eth2-switch-eth1 of the receiving host in Fig. 3. The data from the two series of measurements carried out have been gathered in memory and processed later, using the algorithm of the service curve estimation described in subsection C. The results of these calculations are visualized in Figs. 4, and 5, for the corresponding cases listed in points 1, and 2 above.

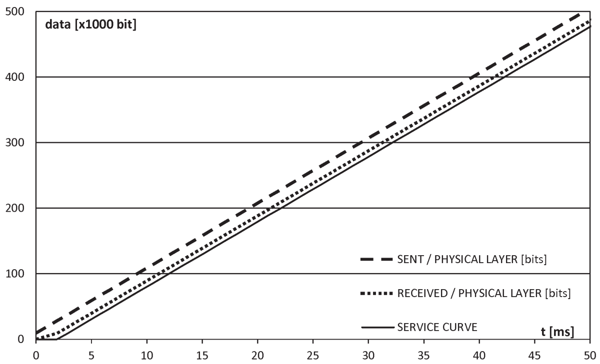

Fig. 4. The cumulative input and output traffic, and the estimated service curve in the case 1.

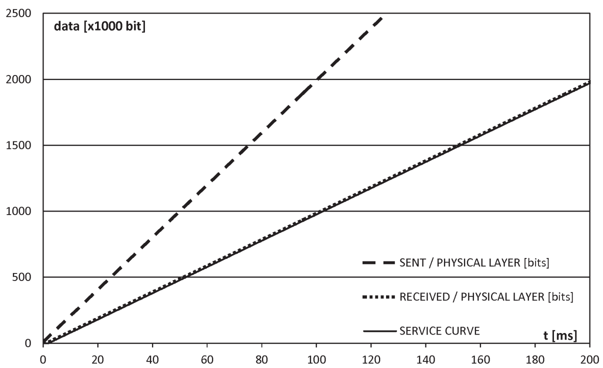

Fig. 5. The cumulative input and output traffic, and the estimated service curve in the case 2.

Comparison of the curves shown in Figs. 4, and 5 with the theoretical ones presented in subsections A and B leads to the conclusion that the measured curves look like similar as predicted by the theory. More results of experiments can be found in [4].

V. Conclusion

Good understanding of network calculus theory is very important for use of this method by network designers. In this paper, we have reviewed some of its fundamentals. Specifically, we have discussed in detail the definition of a greedy shaper, and refined it by completing it with a new theorem, being an extended version of the existing one. In the second part of the paper, we have presented very illustrative examples of network service curve calculations and measurements carried out on a simple teletraffic system. We have analysed and compared the results achieved, specifically concentrating on their agreement with the theory, and concentrating on better understanding of the notion of service curve, too.

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