The Positioning Accuracy Based on the UWB Technology for an Object on Circular Trajectory

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Abstract—This paper presents the comparison of filtering methods – median filtration, moving average Kalman filtration and filtration based on a distance difference to determine the most accurate arm length for circular motion, as a model of wind turbine propellers movement. The experiments have been performed with the UWB technology system containing four anchors and a tag attached to 90cm arm that was rotated with speed up to 15.5 rad/s (as a linear speed of 50km/h). The trilateration concept based on the signal latency has been described in order to determine the position of an object on circular trajectory. The main objective is the circle plane rotation (parallel and perpendicular) with respect to the anchors plane reference system. All research tasks have been performed for various cases of motion schemes in order to get the filtering method for object in motion under best accuracy goal. Filtration methods have been applied on one of two stages of the positioning algorithm: (1) on raw data got from the single anchor-tag (before trilateration); (2) on the position obtained from four anchors and tag (after trilateration). It has been proven that the appropriate filtering allows for higher location accuracy. Moreover, location capabilities with the use of UWB technology – shows prospective use of positioning of objects without access to other positioning forms (ex. GPS) in many aspects of life such as currently developing renewable, green energy sources like wind turbines where the circular motion plays an important role, and precise positioning of propellers is a key element in monitoring the work of the whole wind turbine.

Keywords—Ultra-Wideband, UWB, localization, propellers positioning, trilateration, location filtration

I. INTRODUCTION

In a world that cares for environmentally friendly solutions and renewable energy sources, wind farms play a significant role. Circular motion occurring in wind turbines is natural here, and tracking the propellers can help in monitoring work and performing maintenance-free revisions. However, such a solution requires appropriate technologies and tailor-made algorithms. The location of an object in motion especially when object is moving around a circle is not a trivial task and therefore it should be investigated with special attention.

Determination of objects positions can be done using many different techniques, they can be based on the signal strength indicator, wave propagation latency, phase shift, etc. [5], [6], [7]. There are also many factors that need to be met when comparing locating systems. The most important metrics are: accuracy, availability, coverage area, scalability, cost and security, but it can also specify robustness, integrity, update rate, infrastructure, intrusiveness, approval, market maturity, or number of users [8] [3].

To position objects on medium distances, we can use one or more of these technologies: Wireless Local Area Network (WLAN) [9], Cellular Networks[10], Bluetooth [11], [12], Radio Frequency Identification (RFID) [13], Infrared (IR), Ultrasonic, ZigBee, Image Based Technologies, Pseudolites [14] and the Ultra-Wideband (UWB) [15]. All those techniques can be combined with micro electro-mechanical system (MEMS) sensors [16], [17]. This paper focused on Ultra-Wideband, as a most promising technology, which allows objects to be tracked with centimeters accuracy [18].

Many papers focus on selected metrics [19] and the most common approach is people location [20]. In the case of people, there is weak pressure on high precision [21,22] and computational costs [23] and time to first fix are not critical [24], so it is difficult to look for a solution capable of positioning fast-moving propellers.

However, the proposed solution is not only suitable for positioning propellers, but also can be implemented in a relatively low cost devices (with limited computation resources) like unmanned aerial vehicles [25], indoor transportation systems as biomedical, and hospitals cargo, patients monitoring [26] or storage facilities, smart vacuum cleaners, and the increasingly popular small personal electric vehicles as boards, hoverboards or scooters in a smart cities [27,28]. It can also replace current solutions like determining coarse positioning of mobile devices that play a central role in a Smart City, through advertising purposes [1] and using GPS system to navigate systems and to ADAS (Advanced Driver Assistance Systems [2]) as a full stack real time car positioning system.

The authors present a research on easy implementable and low computations cost filtering methods for an object in circular motion with speed up to 50km/h. The current literature does not touch the subject of determining the convergence of surfaces, in contrast to method presented in this article, that allows to determine the length of the arm and the way it goes through with a very good result.

II. LOCATING SYSTEM DESCRIPTION

This section presents fundamental and essential information about the system applied. The system has been developed by the team based on the resources available on the decaWave web page [29]. The system is composed of 4 anchors and a Tag (see fig. 1). The system has been built with STM32 Discovery development boards and modules DWM1000. It operates across a wide variety of markets, including ePOS and retail, automotive, agriculture, building control and...
automation, factory automation, healthcare, safety and security, warehousing and logistics and a range of others.

A. Description of the equipment and the experiment

The test stand environment consists of a UWB tag that was receiving and processing signals, as well as four UWB anchors serves as a station with a fixed declared position (fig. 1). All devices are integrated with DWM module consisted of decaWave devices DWM1000 that is an IEEE802.15.4-2011 UWB compliant wireless transceiver. As decaWave declare DWM1000 module are optimized for unrivalled (up to 10 cm) indoor precision location and high communications data rate (up to 6.8 Mb/s) for Real Time Location Systems (RTLS) and Wireless Sensor Networks (WSN). Test stand has been composed of four anchors that were placed on a square with a side length of 500 cm, as shown in fig. 2.

The tag has been attached to the arm of length 100 cm. The complete arm has been attached to an engine which was responsible for fast movement. The idea of the rotation scheme has been presented in the fig. 3. The tag antenna was connected 10 cm from the edge of the arm.

The test stand allows for reaching the speed up to 50 km/h (the average speed during the experiment is 50 km/h). It has been assumed two test scenarios: first, the rotation was parallel to the line through anchors 2 and 3 (perpendicular to the line through anchors 1 and 2, towards 2 and 3); second, the rotation was parallel to the line through anchors 1 and 2 (perpendicular to the line through anchors 2 and 3, towards 1 and 2). The fixed point of an engine in 3D environment was at point X,Y,Z (250, 250, 200). The exemplary case has been shown in the fig. 4.

B. Description of the acquired data

During the experiment, two test series were carried out:
1. Label $E_1$ - a test at XYZ {250,250,200} with a propeller running parallel to the line through anchors 2 and 3 (perpendicular to the line through anchors 1 and 2, towards 2 and 3)
2. Label $E_2$ – a test at XYZ {250,250,200} with a propeller moving parallel to the line through anchors 1 and 2 (perpendicular to the line through anchors 2 and 3, towards 1 and 2)

Each series consisted of frames - $F_i$ (for $i=1…N$) containing information about time, marker distance, signal quality and RSSI. The example frame is presented as follows:
\[ F_i = \{47172911; 188; 326; 325; 187; -73.31; -73.55; -76.53; -0.08; 6.95; 14.55; 6.75; 13.04;\} \]

The frame is interpreted in the following manner:

\[ F_i = \{TS; S_1; S_2; S_3; S_4; SS_1; SS_2; SS_3; SS_4; Q_1; Q_1; \ldots; Q_i\} \]

\[ E = \{F_1, F_2, F_3, \ldots, F_n\} \]

Where:

- \( N \) – Total number of data received from the UWB locating system
- \( TS \) - time stamp (it is counted from the system start)
- \( S_1 \) – \( S_4 \) – measured distances between Tag and the following Anchors (in cm)
- \( SS_1 \) – \( SS_4 \) – signal strength indicator for the following anchors with respect to the Tag (in dBm)
- \( Q_1 \) – \( Q_4 \) – quality indicator for all anchors; no unit, values between 0 and 1 – the best quality

The tag distance to a reference point (an anchor) for a test scenario (e.g. label \( E_1 \)) can be represented by the graph, which is shown in fig. 6.

![Fig. 6. The rotation of the tag with respect to anchor no 1 for test scenario E1](image)

Full circles have been selected from the whole dataset (for both test scenarios) based on the local maxima. A set of single circles has been obtained (pay attention it is raw data obtained from frame \( E \)):

\[ C_E = \{C_{11}, C_{12}, C_{13}, C_{14}, C_{21}, C_{22}, C_{23}, C_{24}, \ldots, C_{L1}, C_{L2}, C_{L3}, C_{L4}\} \]

Where: \( E \) – is a test scenario label and \( C_{ij} \) - Upper index \( j \) represents anchor number, here 1, ..., 4; lower index \( i \) represents data set for following circles, here from 1 to \( L \).

### C. The trilateration method

The trilateration algorithm has been applied in order to convert the UWB data to a particular position. At the hardware level a TDOA (Time Difference Of Arrival) was implemented. Communication schema is presented in the [14]. Then, the obtained time, also at the hardware level converted to the distance between particular anchors, and tag, that returns distances in centimetres between the tag and following anchors.

The time of flight \( t_{prop} \) can be simply calculated with the use of the \( (1) \). When the time of flight is known the distance \( d \) between anchor and tags can be calculated with the use of the \( (2) \).

\[ t_{prop} = \frac{t_{round} - t_{reply}}{2} \]

\[ d = v \cdot t_{prop} \]

Where \( v \) – is the speed of light (299792458 [m/s])

The system (anchor no 1) returns distances in centimeters between the tag and following anchors (datagram \( F_i \)). The time stamp should be solid for a single data acquisition – fortunately such requirement is ensured by hardware that performs data acquisition and returns them in packages at specific time intervals.

The principle of operation flows from the fundamental geometry and the main idea is depicted in the fig. 8 [30]. The input data are three (from four available – selection based on RSSI factor) reference points \( A_1 \) (\( x_1, y_1, z_1 \)), \( A_2 \) (\( x_2, y_2, z_2 \)), \( A_3 \) (\( x_3, y_3, z_3 \)) and three distances \( S_1, S_2, S_3 \) to point \( T \). To find the coordinates of the point \( T \) (\( x, y, z \)) is equivalent with the determination of the coordinates of the system of quadratic equations shown in \( (3) \).

![Fig. 8. Trilateration – an example of the operation given for the point - the tag T, and for the three distances S1, S2 and S3 from the three reference points P1, P2 and P3](image)

\[ \begin{align*}
(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 &= S_1^2 \\
(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 &= S_2^2 \\
(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 &= S_3^2
\end{align*} \]

Equation \( (3) \) can be arranged as \( (4) \).

\[ \begin{align*}
(x^2 + y^2 + z^2) - 2x_1x - 2y_1y - 2z_1z &= S_1^2 - x_1^2 - y_1^2 - z_1^2 \\
(x^2 + y^2 + z^2) - 2x_2x - 2y_2y - 2z_2z &= S_2^2 - x_2^2 - y_2^2 - z_2^2 \\
(x^2 + y^2 + z^2) - 2x_3x - 2y_3y - 2z_3z &= S_3^2 - x_3^2 - y_3^2 - z_3^2
\end{align*} \]
Or in matrix representation (5)
\[
\begin{bmatrix}
1-2x_1-2y_1-z_1 \\
1-2x_2-2y_2-z_2 \\
1-2x_3-2y_3-z_3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \begin{bmatrix} x^2 + y^2 + z^2 \\
2x_1y_1 \\
2x_1z_1 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \begin{bmatrix} 0 \\
-2y_1 \\
-2z_1 \\
\end{bmatrix}
\]

Thus equation (5) is represented in the form (6).
\[
B_0 \cdot x = b_0
\]  

(6)

With the constraint
\[
x \in E = \left\{ (x_0, x_1, x_2, x_3)^T \mid x_0 = x_1^2 + x_2^2 + x_3^2 \right\}
\]

In our case, P_1, P_2 and P_3 do not lie on a straight line so the \( \text{Rang}(B_0) = 3 \) and \( \text{dim}(\text{Kern}(B_0)) = 1 \). The general solution of (6) is then equation (8).
\[
x = x_p + t \cdot x_h
\]  

(8)

Where \( t \) is the real parameter, \( x_p \) is a particular solution of equation (8) and \( x_h \) is a solution of homogeneous system of equations (9), i.e. that is a Basis of Kernel \( (B_0) \).
\[
B_0 \cdot x = 0
\]  

(9)

The \( x_p \) and \( x_h \) vectors can be determined using the Gaussian elimination method. The particular solution \( x_p \) can also be excluded by the pseudo inverse of the matrix \( B_0 \).

To determine the parameter \( t \) let do (10).
\[
x_p = (x_{p0}, x_{p1}, x_{p2}, x_{p3})^T \\
x_h = (x_{h0}, x_{h1}, x_{h2}, x_{h3})^T \\
x = (x_0, x_1, x_2, x_3)^T
\]

After inserted equations (10) through (8) we obtain (11).
\[
\begin{align*}
x_0 &= x_{p0} + t \cdot x_{h0} \\
x_1 &= x_{p1} + t \cdot x_{h1} \\
x_2 &= x_{p2} + t \cdot x_{h2} \\
x_3 &= x_{p3} + t \cdot x_{h3}
\end{align*}
\]  

(11)

Where we still using the constraint \( x \in E \) it follows (12).
\[
x_{p0} + t \cdot x_{h0} = \left( x_{p1} + t \cdot x_{h1} \right)^2 + \\
\cdot \left( x_{p2} + t \cdot x_{h2} \right)^2 + \left( x_{p3} + t \cdot x_{h3} \right)^2
\]

And thus (13).
\[
t^2 \left( x_{h1}^2 + x_{h2}^2 + x_{h3}^2 \right) + \\
\cdot \left( 2 \cdot x_{p1} x_{h1} + 2 \cdot x_{p2} x_{h2} + 2 \cdot x_{p3} x_{h3} - x_{h0} \right) + \\
\cdot x_{p1}^2 + x_{p2}^2 + x_{p3}^2 - x_{p0} = 0
\]  

(13)

This is a quadratic equation in the form \( at^2 + bt + c = 0 \) with the solution as (14).
\[
t_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]  

(14)

The solutions of the equation system (6) are (15).
\[
x_1 = x_p + t_1 \cdot x_h \\
x_2 = x_p + t_2 \cdot x_h
\]  

(15)

So in the case of 3D positioning using 3 anchors, the position of \( T_n \) (XYZ) can be represented as \( x_i \) (XYZ) or \( x_o \) (XYZ) depending on the expected range of positions in the X Y Z axes.

The choice of the right position was arbitrarily made as a return item in parameter \( x_i \) due to the significant difference between \( x_i \) and \( x_o \) positions, as well as the high distance from the border of those areas that could overlap the spheres received from the UWB system.

D. Filtration methods

As mentioned before, the raw data from the system needs filtration due to noise and environment conditions. For this reason, the following filtrations have been applied on the first or second stage (for both stages if indicated). First stage raw data means distances between an anchor and the tag (frame F), second stage raw data means 3D position after trilateration (tag position \( T \)). The following filters have been investigated:

- The median filter. In the research the filter has been applied with the following window size: 1 (as no filtration), 3, 5, 9 and 11, before, and after trilateration. Median with window size \( k \) of \( N_i \) samples, where \( i = 1, \ldots, k \) and \( N_i \leq N_i + 1 \) is presented in equation (16).
\[
\text{med}(N_i) = \left\{ \begin{array}{ll}
N_{i+1} & \text{for } k = 2j + 1 \\
\frac{1}{2}(N_i + N_{i+1}) & \text{for } k = 2j
\end{array} \right.
\]  

(16)

In this case, the value was always an odd number, so the \( k = 2j + 1 \) was used.

- Kalman filtering (KF), (also known as linear quadratic estimation - LQE), is an recursion algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement along [31,32].

The Kalman filter model assumes the true state at time \( k \) is evolved from the state at \( (k - 1) \) (17).
\[
x_k = Ax_{k-1} + w_{k-1} \\
z_k = Cx_{k-1} + v_{k-1}
\]  

(17)

The classical form of a linear-discrete KF is given by prediction shown as (18).
\[
\bar{x}(k|k-1) = A \bar{x}(k-1|k-1) \\
P(k|k-1) = A \bar{P}(k-1|k-1)A^T_C + Q_{k-1} \\
v_k = z_n - C \bar{x}(k|k-1) \\
S_k = C \bar{P}(k|k-1)C^T_k + R
\]  

(18)

And the filtering process in equations (19).
THE POSITIONING ACCURACY BASED ON THE UWB TECHNOLOGY FOR AN OBJECT ON CIRCULAR TRAJECTORY

\[ K_k = P(k+1) - C_k^2 S_k \]
\[ \hat{x}(k) = \hat{x}(k-1) + K_k e_k \]
\[ P(k) = (I_k - K_k C_k) P(k-1) \]

- Moving average types: simple and triangular with window sizes: 1 (as no filtration), 3, 5, 9 and 11 before, and after trilateration.

First, there is calculated the simple moving average (SMA) (20).

\[ \text{SMA}(T_j) = \frac{1}{k} \sum_{j=-\frac{k}{2}}^{\frac{k}{2}} T_j \]

Then, take the average of all the SMA values to get TMA (triangular moving average) values (21).

\[ \text{TMA(SMA)}_j = \frac{1}{k} \sum_{j=-\frac{k}{2}}^{\frac{k}{2}} \text{SMA}_j \]

- Pre-processing for discarding points if the current position is further with respect the previous one under maximum speed condition. The subsequent data analysis have been applied for the individual measurement series – for filtration before trilateration for point T and after trilateration for \( X_T, Y_T, Z_T \) as a current point (as obtained in II C), and \( J \) as a filter size the condition (22) is checked.

\[ |X_T - X_{T+1}| < J \land |Y_T - Y_{T+1}| < J \land |Z_T - Z_{T+1}| < J \] (22)

If the above statement is true then the point is further processed. Otherwise (23)

\[ X_{T+1} = X_T \land Y_{T+1} = Y_T \land Z_{T+1} = Z_T \land i = i + 1 \] (23)

A. Validation methods

The whole process for object localization should be verified with a unified metric. The authors propose validation for the circle projection in two dimensions, z axis is discarded. In order to make projection on the plane, a circle was observed from a point perpendicular to the center of the circle (X Y Z as 250 cm 500 cm 200 cm) with an azimuth equal to 90 and an elevation equal to 0. The following methods of verification are proposed:

- Area of the circle described at the points - used to compare the maximum measurement error outside the test area, relative to the expected area of the circle. Firstly for each points \( i, j \in N \), where \( N \) was number of all the distance between the furthest points, and the \( p_{\text{near}} \) and \( p_{\text{far}} \) as the furthest points was determined (24).

\[ p_{\text{near}} = N_{x, y} \land p_{\text{far}} = N_{x, y} \leftrightarrow \max(\text{dist}) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \] (24)

From the distance the \( r \) was obtained (25) and the center of the circle – \( O(x, y) \) (26).

\[ r = \frac{\text{dist}}{2} \]
\[ O(x, y) = \frac{p_{\text{near}} + p_{\text{far}}, p_{\text{near}} + p_{\text{far}}}{2} \] (26)

- Area of circle fitted to the points by the least squares method - used to compare the measurement error of the circle seen straight ahead of the expected area of the circle. For two dimensional matrix \( M \) of \( X_k, Y_k \) the mldivide operation that solve systems of linear equations with \( (x^2, y^2) \) was performed, next the center of the circle values \( S_x \) and \( S_y \) values was obtained as a \( O_x = 0.5 * M(1), O_y = 0.5 * M(2) \), and the radius (27)

\[ r = \sqrt{\frac{O_x^2 + O_y^2}{4} - a_3} \] (27)

- Area of ellipse fitted to the points by the least squares method - used to compare the measurement error of the circle seen from the perspective of the observer, relative to the expected area of the circle.

- Area of polygon escribed at the points - Used to compare the measurement error of the received polygon in relation to the expected area of the circle. To get the polygon, the convex hull was used. The convex hull of a set of points \( O \) in n dimensions is the intersection of all convex sets containing \( O \). For \( N \) points \( p_1, ..., p_N \), the convex hull \( C \) is then given by the expression (28)

\[ C = \left\{ \sum_{j=1}^{N} \lambda_j p_j; \lambda_j \geq 0 \text{ for all } j \text{ and } \sum_{j=1}^{N} \lambda_j = 1 \right\} \] (28)

- The circle area is calculated by using equation (29).

\[ AR_{\text{circle}} = 2 \int_{-r}^{r} \sqrt{r^2 - x^2} \, dx \] (29)

- The ellipse area is calculated for a as a half of longer radius and b as a half of shorter radius by using equation (30):

\[ AR_{\text{ellipse}} = 4 \int_{0}^{a} b \sqrt{1 - \frac{x^2}{a^2}} \, dx \] (30)

- The polygon area is calculated by using equation (31).

\[ AR_{\text{polygon}} = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) \] (31)

III. Results and Discussion

The E1 data set was divided into C1 circles, where \( i = 1, ..., 87 \) and the E2 data series was divided into C2 circles, where \( i = 1, ..., 115 \) which have been subjected to filtration. Selected results for median filtration combined with the results obtained for ellipse area are as follows:
The highest uniformity of readings for the median filter during observation of the ellipse is obtained for filtration with window size 5 before trilateration and without filtration on vectors T. Average surface area based on experiment E1, i.e. 87 circles in this point was 1.872 m² what is equivalent of 77 cm long arm (inaccuracy equals 13 cm). However it might be noticed, that similar results - 0.011 gives no filtration, and filtration with window 3 and 5 before trilateration without filtration after trilateration. Each of them give similar accuracy. Also noticeable is the decreasing surface with increasing filter window, thus narrowing the circle to the center.

For the second experiment E2, the number of circles are 115. The best result occurs for filtration with window size 5 before trilateration, i.e. 0.025 with ellipse area 2.439 m², what gives 88 cm – difference with expected value is 2 cm. As we can see in the fig. 9 visual difference with unfiltered circle (A) and filtered with windows size 5 before trilateration (B) is very small. Difference is in the (C) example was obtained with filter window 7 before, and after trilateration. The arm length calculated from the obtained area in this case is for (A) 86 cm for polygon, 100 cm for circle escribed on points, 85 cm for circle fitted to the points and 85 cm for ellipse fitted to the points. Next for the (B) result is analogous: polygon – 86, circle escribed – 100 cm, circle fitted – 85 cm, ellipse fitted – 85 cm. And for the (C): polygon – 78 cm, circle escribed – 95 cm, circle fitted – 81 cm, ellipse fitted – 79 cm.

Fig. 9. Filtration results for E2: A – raw data, B – median filter: size 7 for C9 data and size 1 on T9; C – median filter: size 7 for C9 data and size 7 on data T9

Next was performed test based of moving average. Again, the results obtained for the ellipse are presented.

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<th>5</th>
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<td>0.025</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.049</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Table I
Variance of ellipse surface area for 87 circles from data series E1: filtered by median filter with step from 1 to 11 for C and T data.

Table II
Variance of ellipse surface area for 87 circles from data series E1: filtered by moving average with step from 1 to 11 for C and T data.

In case presented in table II, variance of ellipse surface area is smallest also for filtration on C vectors with window size 7 without filtration on T vectors, which corresponds to an average surface area of 1.854 m². That is also equivalent of 77 cm long arm, what like in the previous example give 13 cm inaccuracy. Second smallest result is for filtration with window 5 for C, and 3 for T, what gives 1.814 m² – average area in this case is equivalent of 76 cm long arm.

For the E2 data series minimal variance occurred for filtration with window size 5 on C vector – 0.013 with average surface area 2.272 m² what is equivalent of 85 cm long arm 5 cm difference from the expected value. Also the circle after filtration with window size 3 on vectors C give in this example similar result – 2.272 m².

Kalman filtration was investigated at the third stage. Results are shown in Table III.

Table III
The values of statistics for measurement with the Kalman filtration for E2 data series on T vectors

<table>
<thead>
<tr>
<th></th>
<th>Polygon escribed on points</th>
<th>Circle escribed on points</th>
<th>Circle fitted to the points</th>
<th>Ellipse fitted to the points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average [area m²]</strong></td>
<td>2.462</td>
<td>3.749</td>
<td>2.575</td>
<td>2.699</td>
</tr>
<tr>
<td><strong>Arm length [cm]</strong></td>
<td>89</td>
<td>109</td>
<td>91</td>
<td>93</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.007</td>
<td>0.042</td>
<td><strong>0.009</strong></td>
<td>0.019</td>
</tr>
<tr>
<td><strong>Median [area m²]</strong></td>
<td>2.459</td>
<td>3.775</td>
<td>2.576</td>
<td>2.703</td>
</tr>
<tr>
<td><strong>Arm length [cm]</strong></td>
<td>88</td>
<td>110</td>
<td>91</td>
<td>93</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>0.085</td>
<td>0.205</td>
<td>0.093</td>
<td>0.137</td>
</tr>
<tr>
<td><strong>Max [area m²]</strong></td>
<td>2.668</td>
<td>4.422</td>
<td>2.794</td>
<td>2.985</td>
</tr>
<tr>
<td><strong>Arm length [cm]</strong></td>
<td>92</td>
<td>119</td>
<td>94</td>
<td>97</td>
</tr>
<tr>
<td><strong>Min [area m²]</strong></td>
<td>2.265</td>
<td>3.138</td>
<td>2.300</td>
<td>2.179</td>
</tr>
<tr>
<td><strong>Arm length [cm]</strong></td>
<td>85</td>
<td>100</td>
<td>86</td>
<td>83</td>
</tr>
</tbody>
</table>

In this case from the Kalman filtration statistic can be read, that in case of ellipse, but also circle fitted to the points obtained results are most close to the expected ones – respectively 2.699 m² for ellipse and 2.575 m² for fitted circle what gives 0.93 cm and 0.91 cm long arm. This means that the manufacturer’s declared accuracy of up to 10 cm of moving objects up to 18 km/h has been improved up to 3 cm in speed at 50 km/h at a distance of 250 cm from the reference antenna placed at the same height, when we use ellipse fitted to the points, and gives 1 cm when we use circle fitted to the points. Also, the worst case from whole test for circle fitted to the...
points gives 94 cm and 86 cm long arm (4 cm difference from declared), and worst case for the ellipse gives 97 cm and 83 cm long arm what means 7 cm difference from declared length. Another pross of using Kalman filtration in this case has also very low variance – 0.009 for fitted circle and 0.019 for ellipse.

Results of filtering first data series - E₁, using the Kalman filter are presented in the table IV. It can be seen that areas of polygon escribed on points, as well as circle and ellipse fitted to the points are very similar to each other. All three surface areas correspond to the 79 cm – 82 cm long arm. Also for these three values, the biggest inaccurate occur to polygon escribed on the points and is 16 cm. The circle escribed on points, which is the validator of the maximum error for a single dataset, in its maximum value is 3.660 m² what is corresponding to the 108 cm long arm – difference from the expected value is 18 cm.

TABLE IV
THE VALUES OF STATISTICS FOR MEASUREMENT WITH THE KALMAN FILTRATION AFTER TRILETRATION FOR E₁ DATA SERIES

<table>
<thead>
<tr>
<th></th>
<th>Polygon escribed on points</th>
<th>Circle escribed on points</th>
<th>Circle fitted to the points</th>
<th>Ellipse fitted to the points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average [area m²]</td>
<td>1.967</td>
<td>3.380</td>
<td>2.095</td>
<td>2.021</td>
</tr>
<tr>
<td>Arm length [cm]</td>
<td>79</td>
<td>104</td>
<td>82</td>
<td>80</td>
</tr>
<tr>
<td>Variance</td>
<td>0.008</td>
<td>0.015</td>
<td>0.005</td>
<td>0.012</td>
</tr>
<tr>
<td>Median [area m²]</td>
<td>1.968</td>
<td>3.379</td>
<td>2.097</td>
<td>2.006</td>
</tr>
<tr>
<td>Arm length [cm]</td>
<td>79</td>
<td>104</td>
<td>82</td>
<td>80</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.088</td>
<td>0.123</td>
<td>0.069</td>
<td>0.111</td>
</tr>
<tr>
<td>Max [area m²]</td>
<td>2.188</td>
<td>3.660</td>
<td>2.555</td>
<td>2.367</td>
</tr>
<tr>
<td>Arm length [cm]</td>
<td>83</td>
<td>108</td>
<td>85</td>
<td>87</td>
</tr>
<tr>
<td>Min [area m²]</td>
<td>1.740</td>
<td>3.133</td>
<td>1.922</td>
<td>1.808</td>
</tr>
<tr>
<td>Arm length [cm]</td>
<td>74</td>
<td>100</td>
<td>78</td>
<td>76</td>
</tr>
</tbody>
</table>

In the fig. 10 below, is presented sample result of filtration using Kalman filter. In the (A) figure – filtration without trilateration - result of arm length calculated basing on surface area is 86 cm for polygon, 100 cm for circle escribed on points, 85 cm for circle fitted to the points and 85 cm for ellipse fitted to the points. For the (B) figure it is 87 cm for the polygon (+1 cm to the expected result relative to data without filtration), 103 cm for the circle escribed on the points (-3 cm), 87 cm for the circle fitted (+2 cm) and 87 for the ellipse (+2 cm).

The last type of tested filtration was filtration based on the difference in distance between points. The result of performed filtration is presented in table V.

TABLE V
VARIANCE OF ELLIPSE SURFACE AREA FOR 87 CIRCLES FROM DATA SERIES E₂ FILTERED BY FILTRATION BASED ON THE DIFFERENCE IN DISTANCE BETWEEN POINTS WITH STEP FROM 1000 (AS NO FILTRATION) DOWN TO 20 FOR BOTH DATA VECTORS

<table>
<thead>
<tr>
<th>Size of filter window [cm] after trilateration (T)</th>
<th>Size of filter window [cm] before trilateration (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 100 50 40 35 30 25 20</td>
<td>1000 100 50 40 35 30 25 20</td>
</tr>
<tr>
<td>0.001 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
<td>0.001 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
</tr>
<tr>
<td>0.01 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
<td>0.01 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
</tr>
<tr>
<td>0.011 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
<td>0.011 0.011 0.011 0.014 0.016 0.011 0.009 0.327</td>
</tr>
<tr>
<td>0.02 0.012 0.011 0.015 0.018 0.023 0.075 0.476</td>
<td>0.02 0.012 0.011 0.015 0.018 0.023 0.075 0.476</td>
</tr>
<tr>
<td>0.012 0.012 0.014 0.016 0.018 0.025 0.084 0.925</td>
<td>0.012 0.012 0.014 0.016 0.018 0.025 0.084 0.925</td>
</tr>
<tr>
<td>0.016 0.016 0.024 0.056 0.074 0.181 0.213 1.057</td>
<td>0.016 0.016 0.024 0.056 0.074 0.181 0.213 1.057</td>
</tr>
<tr>
<td>0.183 0.230 0.517 0.430 0.534 0.609 1.329 0.786</td>
<td>0.183 0.230 0.517 0.430 0.534 0.609 1.329 0.786</td>
</tr>
</tbody>
</table>

In this method we calculate the difference between next points, and if the difference was larger than size of filter window, the point was replaced with previous one. For filtration on C vectors filtration was subjected to the distance between tag and the individual reference points. For filtration on T vectors, the linear distance on the X and Y axis was independently tested. What we can see in the table, is that the filtration on T with window size 25 gives smallest variance – 0.009 what corresponds with surface area 1.925 m², 1.925 m² and 1.921 m² for filter window size 1000 cm – as no filtration, 100 cm and 50 cm. These distances are equivalent of an arm of length 78 cm – 12 cm difference from declared 90 cm. For the E₂ data series smallest variance for the fitted circle was obtained for filtration window 40 cm on vector F and 1000 cm and 100 cm for T what gives area 2.319 m² and arm length 86 cm – 4 cm difference from the expected value.

IV. CONCLUSION

Localization of objects in circular motion with the use of UWB technology gives high accuracy in relatively short time. The paper shows influence of chosen filters on localization accuracy under predefined metrics: circle and ellipse, and the radius of real arm which have been compared with one obtained after filtration. In details, median filter accuracy is in range 2% - 14% (i.e. 2 cm – 13 cm) depending on the selected type of motion and samples axis. Moving average filter is less accurate than median filter because the absolute radius minimum error is 6% (i.e. 5 cm). Distance-based filtration accuracy deviates form 6% up to 8% (i.e. 5 cm – 7 cm). The best results give filtration based on Kalman filter – its accuracy is improved from declared 11% (10 cm) up to 1% (1 cm), so the length of the arm is almost equal to the real one.

On the basis of the tests, it can be said that the precision in determining the required length of the arm depends on many factors (speed, position of the observer, type of filtration, surface of motion with respect to anchors). The proposed
metrics, i.e. obtained from the surface area, are well suited to reflect the movement of the UWB tag during the measurement series. The research schemes investigated in this paper allows for object positioning and object localization with good enough accuracy. It can be applied location of wind turbine propellers for monitoring purposes, but also for indoor and outdoor positioning cases. If other systems, like MEMS are available then the accuracy can be increased. For indoor environment, where GPS signal is weak, the system allows for positioning, i.e. indoor navigation. Such system can be introduced in smart cities as cheap alternative system for object positioning within selected areas, e.g. on crossroads where the accident probability is relatively high.

REFERENCES